Math 1231 Fall 2022
Single-Variable Calculus I Section 16
Mastery Quiz 3
Due Monday, September 19

This week's mastery quiz has two topics. Everyone should submit both topics. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Monday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 2: The Squeeze Theorem

Name:

Recitation Section:

Major Topic 1: Computing Limits

Compute the following limits, justifying your answers with clean and correct work.

(a) $\lim_{x\to 3} \dfrac{\sqrt{7-x}-2}{x-3} =$

Solution:

$$\lim_{x\to 3} \frac{\sqrt{7-x}-2}{x-3} = \lim_{x\to 3} \frac{7-x-4}{(x-3)(\sqrt{7-x}+2)} = \lim_{x\to 3} \frac{-1}{\sqrt{7-x}+2} = \frac{-1}{4}.$$

(b) Compute $\lim_{x\to 0} \dfrac{\sin(3x)\sin(4x)}{x\sin(2x)} =$

Solution:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)} &= \lim_{x\to 0} \frac{\sin(3x)}{3x}\frac{\sin(4x)}{4x}\frac{12x^2}{x\sin(2x)} \\ &= \lim_{x\to 0} \frac{2x}{\sin(2x)}\frac{6x}{x} \\ &= \lim_{x\to 0} \frac{6x}{x} = 6.\end{aligned}$$

(c) $\lim_{x\to 3} \dfrac{1-x}{(x-3)^3} =$

Solution: The top approaches -2 and the bottom approaches 0, so

$$\lim_{x\to 3} \frac{1-x}{(x-3)^3} = \pm\infty.$$

Since the denominator can be either positive or negative, we can't be any more precise than this.

Secondary Topic 2: Squeeze Theorem

Show that $\lim_{x\to 2}(x-2)\left(1+\sin\left(\dfrac{1}{x-2}\right)\right) = 0.$

Solution: We know that

$$\begin{aligned} -1 &\leq \sin\left(\frac{1}{x-2}\right) \leq 1 \\ 0 &\leq 1 + \sin\left(\frac{1}{x-2}\right) \leq 2 \\ 0 &\leq \left|(x-2)\left(1+\sin\left(\frac{1}{x-2}\right)\right|\right) \leq 2|x-2|. \end{aligned}$$

Since $\lim_{x\to 2} 0 = 0$ and $\lim_{x\to 2} 2|x-2| = 0$, by the Squeeze Theorem, we know that $\lim_{x\to 2}(x-2)\left(1+\sin\left(\frac{1}{x-2}\right)\right) = 0$.