Math 1231: Single-Variable Calculus 1
George Washington University Fall 2022
Recitation 3

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Problem 1. Let $h(x) = \frac{x-1}{\sqrt{5-x}-2}$.

(a) Is this function continuous where it's defined? Where is it undefined?

(b) We can factor an $x-1$ out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the $x-1$ appear. What tricks do we have that might work?

(c) What is $\lim_{X \to 1} h(x)$?

Problem 2. We want to compute $\lim_{x \to 3}(x-3)\left(5\sin\left(\frac{1}{x-3}\right) - 2\right)$.

(a) Is this function continuous where defined? Is it defined at 3?

(b) Does this look like the squeeze theorem might be helpful? Why?

(c) Can you find bounds for the most difficult piece of the function? That is, find a number that's always greater and a number that's always less.

(d) Can we use that to find bounds for $5\sin\left(\frac{1}{x-3}\right) - 2$?

If you haven't already, check in with your neighbors and make sure everyone has the same (or similar) bounds. The next part won't make sense if you have something wrong here.

(e) We want to multiply by $x-3$, but that doesn't work because it might be negative, so now we want to put our work inside absolute value signs. Can we just take the absolute value of all three sides of the inequality you got in (d)?

(f) Can we find upper and lower bounds for $\left|5\sin\left(\frac{1}{x-3}\right)-2\right|$? What has to be true here?

(g) Use those bounds, and the squeeze theorem, to show that

$$\lim_{x\to 3}(x-3)\left(5\sin\left(\frac{1}{x-3}\right)-2\right)=0.$$

Problem 3. We want to compute $\lim_{x\to 3}\frac{\sin(x^2-9)}{x-3}$.

(a) What rule do we know we need to invoke here?

(b) What θ are we going to need to pick for this to work out, and why?

(c) Do algebra so that you can invoke the small angle approximation. What is the limit? (Are you using the AIF property?)

(d) Go back to the beginning, and see what our heuristic idea that $\sin(\theta)\approx\theta$ would have told you. Does that match with what you got?

Problem 4. We want to think about the ways that infinity doesn't really work like a number, and we can't do arithmetic with it.

(a) To start: what is $\lim_{x\to 0} 1/x$, and why?

(b) Let's look at $\lim_{x\to 0} 1/x+1/x$. If we computed the limit of each fraction individually, what indeterminate form would we get?

(c) How do we actually compute $\lim_{x\to 0}\frac{1}{x}+\frac{1}{x}$? (Hint: combine them into one fraction.) Does this make sense in light of what you got in part (b)?

(d) Now consider $\lim_{x\to 0}\frac{1}{x}+\frac{x-1}{x-x^2}$. What is the limit of each piece, and what indeterminate form is this?

(e) Compute $\lim_{x\to 0}\frac{1}{x}+\frac{x-1}{x-x^2}$ directly. Does this make sense in light of what you got in part (d)?

(f) Now consider $\lim_{x\to 0} 1/x+1/x^2$. What indeterminate form would this represent? What is the limit? Do those make sense together?

(g) Finally, let's look at $\lim_{x\to 0}\frac{1}{x}+\frac{x^2-3x+2}{x^2-2x}$. What indeterminate form is this? What is the limit?

(h) What pattern do you see from all of these?