

Math 1231: Single-Variable Calculus 1
George Washington University Fall 2022
Recitation 3

Jay Daigle

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Problem 1. (a) Consider $\lim_{x \rightarrow -\infty} \frac{x}{x+1}$. Can you come up with a heuristic guess about what this limit is?

(b) Can you carefully justify your guess from part (a).

(c) Now consider $\lim_{x \rightarrow +\infty} \frac{x}{\sqrt{3x^2+x}}$, and come up with a heuristic estimate for the limit.

(d) Carefully justify your guess from part (c).

(e) How would either of those calculations change if we take the limit to the other infinity?

Problem 2.

(a) In class we saw that $\lim_{x \rightarrow +\infty} \sqrt{x^2+1} - x = 0$. But what is $\lim_{x \rightarrow +\infty} \sqrt{x^2+x+1} - x$?

(b) What is $\lim_{x \rightarrow +\infty} \sqrt{x^2+ax+1} - x$?

(c) What does the answer in part (b) say about $\lim_{x \rightarrow +\infty} \sqrt{x^2+2x+1} - x$? Why should the answer to this question be obvious?

Problem 3 (Optional). In class we saw that if $f(x) = \sqrt{x}$, then $f'(a) = \frac{1}{2\sqrt{a}}$.

(a) What is $f'(9)$?

(b) Can you use this to estimate $\sqrt{10}$?

(c) Can you use this to estimate $\sqrt{8}$?

- (d) Can you use this to estimate $\sqrt{100}$? Should you?
- (e) Can you use this to estimate $\sqrt{-1}$? Should you?

Problem 4. Let $f(x) = x^3$. We want to find a formula for the derivative of this function at any given point.

- (a) Write down a formula for $f'(a)$ using the $h \rightarrow 0$ limit formulation.
- (b) Use your formula from part (a) to compute the derivative.
- (c) Now write down a formula for $f'(a)$ using the $x \rightarrow a$ limit formulation. Does this look easier or harder than the formula from part (a), and why?
- (d) Use the formula from part (c) to compute the derivative. You should get the same answer you got in part (b).
- (e) Which method was faster? Which method was easier?

Problem 5. Let $f(x) = \sqrt{x^2 - 4}$.

- (a) Set up a limit expression to calculate $f'(x)$. Do you think $h \rightarrow 0$ or $x \rightarrow a$ will be easier here?
- (b) Compute $f'(x)$.
- (c) Where is f differentiable? Where is it not differentiable?