

Math 2233 Fall 2022  
Multivariable Calculus Section 12  
Mastery Quiz 1  
Due Wednesday, September 7

This week's mastery quiz has one topic. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 1: Vectors

**Name:**

**Recitation Section:**

## Major Topic 1: Vectors

- (a) Find a vector from the point  $P = (1, -5, 2)$  to the point  $Q = (-2, 1, 4)$ .

**Solution:** The vector is  $-3\vec{i} + 6\vec{j} - 2\vec{k}$ .

- (b) Let  $\vec{v} = 3\vec{j} - 2\vec{k}$ . If we start at the point  $(2, 1, 1)$  and then follow the vector  $-2\vec{v}$ , where do we end?

**Solution:** We wind up at the point  $(2, 1, 1) - 6\vec{j} + 4\vec{k} = (2, -5, 5)$ .

- (c) Find the orthogonal decomposition of  $\vec{v} = 4\vec{i} + \vec{j} - \vec{k}$  with respect to  $\vec{u} = 2\vec{i} - \vec{j} + 3\vec{k}$ .

**Solution:** First we compute the projection

$$\begin{aligned} \text{Proj}_{\vec{u}} \vec{v} &= \frac{\vec{v} \cdot \vec{u}}{\vec{u} \cdot \vec{u}} \vec{u} \\ &= \frac{8 - 1 - 3}{4 + 1 + 9} \langle 2, -1, 3 \rangle \\ &= \frac{4}{14} \langle 2, -1, 3 \rangle = \frac{4}{7} \vec{i} - \frac{2}{7} \vec{j} + \frac{6}{7} \vec{k}. \end{aligned}$$

Now we still need the perpendicular component, but this is a straightforward subtraction:

$$\begin{aligned} \vec{v}_{\perp} &= \vec{v} - \text{Proj}_{\vec{u}} \vec{v} \\ &= 4\vec{i} + \vec{j} - \vec{k} - \left( \frac{4}{7} \vec{i} - \frac{2}{7} \vec{j} + \frac{6}{7} \vec{k} \right) \\ &= \frac{24}{7} \vec{i} + \frac{9}{7} \vec{j} - \frac{13}{7} \vec{k}. \end{aligned}$$

- (d) Find  $\cos \theta$  where  $\theta$  is the angle between  $\vec{u} = \vec{i} - 2\vec{j} + \vec{k}$  and  $\vec{v} = 5\vec{i} + 2\vec{j} - 6\vec{k}$ .

**Solution:**

$$\begin{aligned} \cos \theta &= \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|} \\ &= \frac{5 - 4 - 6}{\sqrt{6} \sqrt{65}} \\ &= \frac{-5}{\sqrt{390}}. \end{aligned}$$