§1 Transcendental Functions

$$\frac{x^3 - \sqrt[3]{x^2} + 5}{3x-2}$$

algebraic function

$\sin(x)$
$\cos(x)$

transcendental functions

e^x, $\log(x)$,
$\arcsin(x)$,

§1.1 Invertible functions

A fn is a rule

inputs $\rightarrow$ outputs

$f(x) = x+4$ | $f(x) = 7 \Longrightarrow x = 3$

$f(2) = 6$

Dfn: f a fn, $g(f(x)) = x$ for every x

then g is an <u>inverse</u> of f.

$$f(x) = x$$
$$g(x) = x$$

$$f(x) = 5x+3$$
$$y = 5x+3$$
$$y-3 = 5x$$
$$\frac{y-3}{5} = x$$

$$g(y) = \frac{y-3}{5}$$

$$\cancel{g(8) = \frac{8-3}{5}}$$

$$f(x) = x^3$$
$$g(x) = \sqrt[3]{x}$$

write $g(x) = f^{-1}(x)$

finding $f^{-1}(y)$
is solving $f(x) = y$.

$$f^{-1}(y) = x$$

$f(x) = x^2$ not invertible

$f(a) = 9$

$a = ?$

$a = 3$ or -3

$f^{-1}(9) = 3$ or -3

not a fn

Dfn: f is 1-1 or injective if:

whenever $f(a) = f(b)$

then $a = b$.

$f(x) = x^2$ not 1-1

$f(-1) = 1 = f(1)$

$f(x) = x$

suppose $f(a) = f(b)$

$$\begin{array}{ccc} f(a) & = & f(b) \\ \| & & \| \\ a & & b \end{array}$$

So $a = b$.

is 1-1.

$f(x) = \sqrt{x}$

suppose $f(a) = f(b)$

$$\sqrt{a} = \sqrt{b}$$

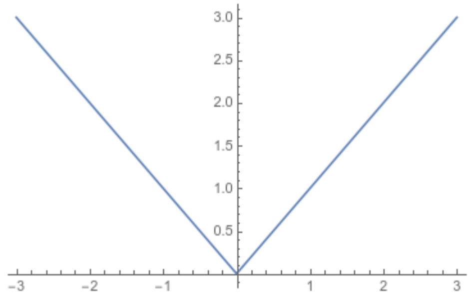
$$\underbrace{(\sqrt{a})^2}_{a} = \underbrace{(\sqrt{b})^2}_{b}$$

this is 1-1.

$f(x) = |x|$

$f(-2) = 2 = f(2)$

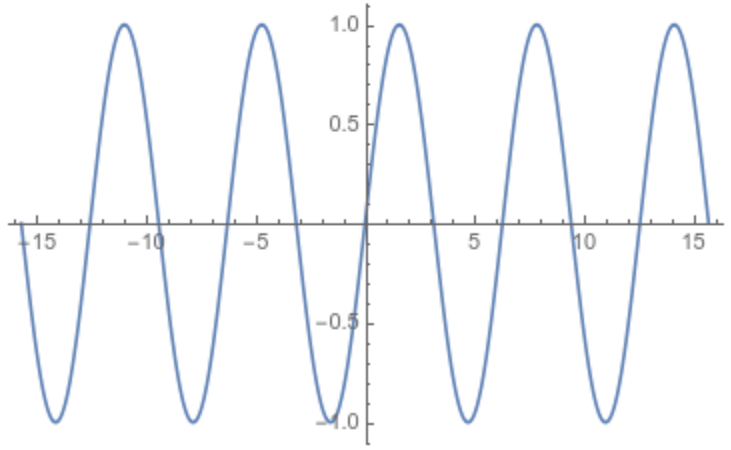
not 1-1



$f(x) = \sin(x)$

$\sin(0) = 0 = \sin(2\pi)$

not 1-1



$f(x) = 3$

$f(\pi) = 3 = f(1700)$

not 1-1

Restricting domain

$f(x) = x^2$ on $[0, +\infty)$

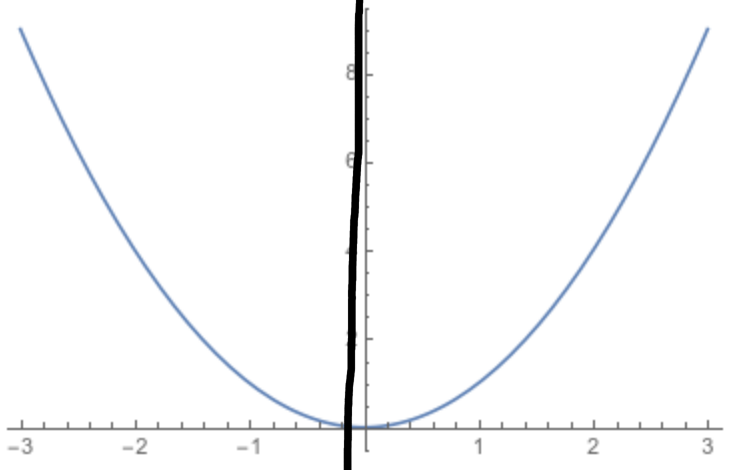
$f(a) = 9 \Rightarrow a = 3$

This is 1-1

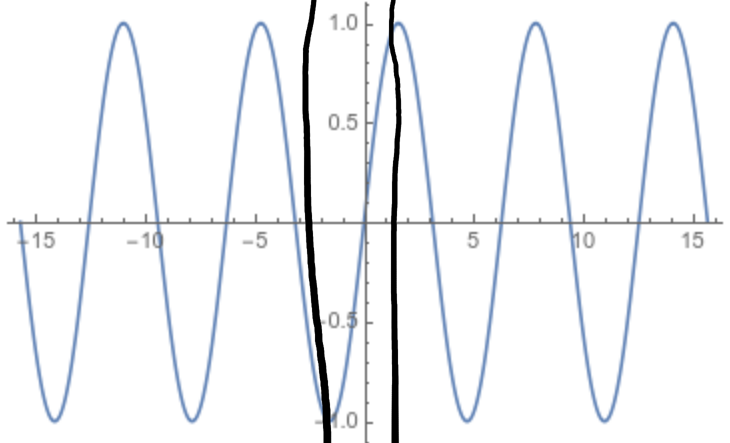
and $f^{-1}(x) = \sqrt{x}$

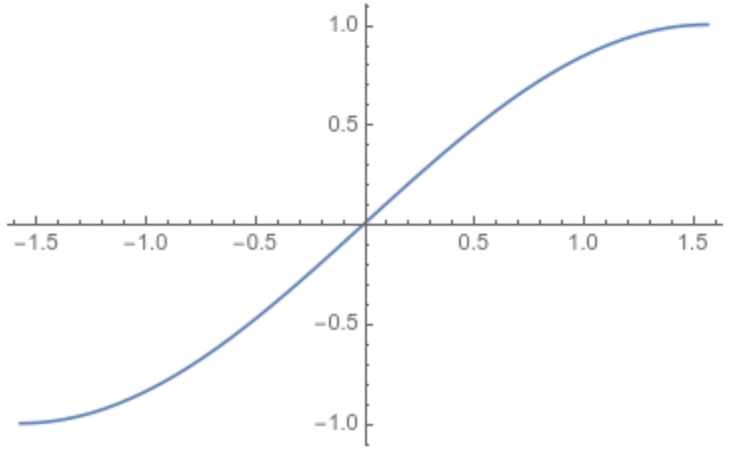
$f(x) = x^2$ on $(-\infty, 0]$,

$f^{-1}(x) = -\sqrt{x}$



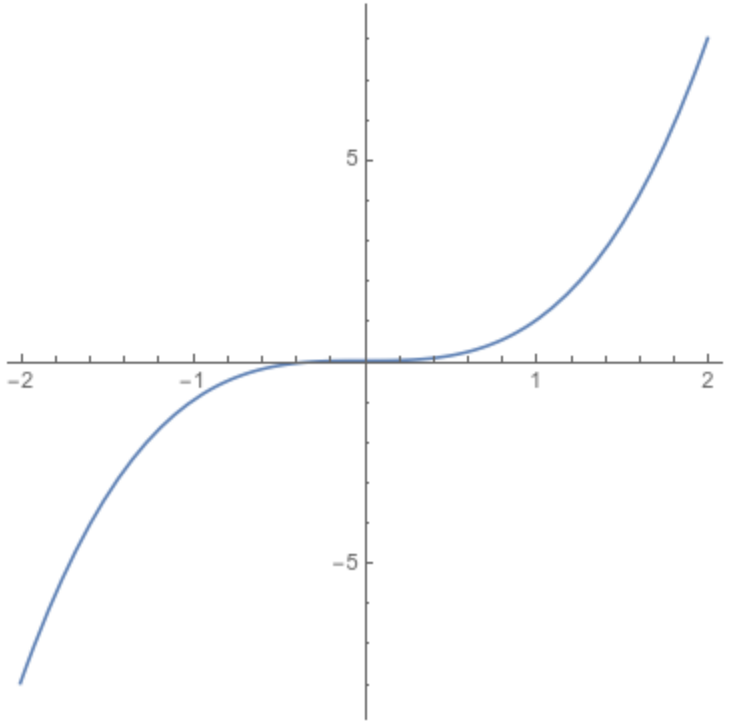
$\sin(x)$



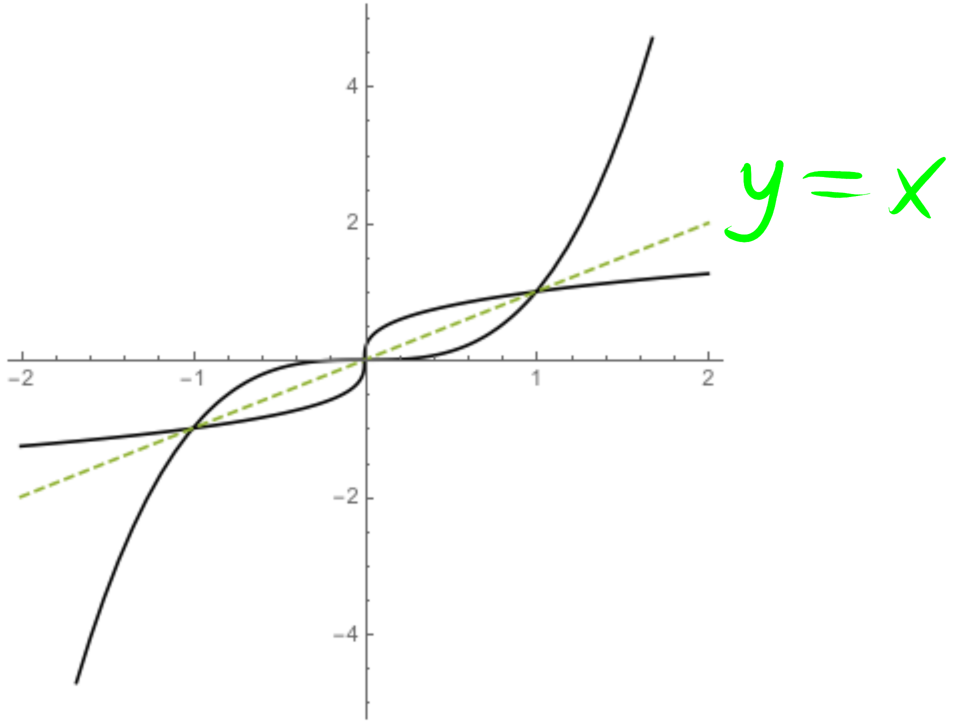


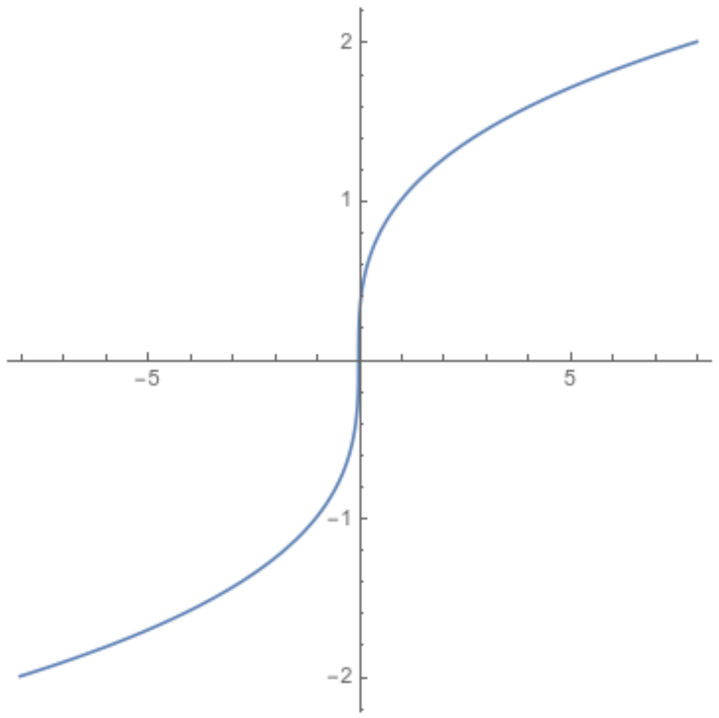
$\sin(x)$ on $[-\pi/2, \pi/2]$ is 1-1.

$f^{-1}(x) = \arcsin(x)$.



x^3





$\sqrt[3]{x}$

Horizontal line test

f is 1-1 $\underbrace{\text{if and only if}}_{\text{iff}}$

any horizontal line hits the graph in at most 1 pt.

Calculus of inverse fns

f is cts iff f^{-1} is cts.

$$x \xmapsto{f} y \qquad \frac{dy}{dx}$$

$$y \xmapsto{f^{-1}} x \qquad \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

$$f^{-1}(f(x)) = x.$$

$$y = f(x)$$

$$x = f^{-1}(y)$$

$$(f^{-1})'(f(x)) \cdot f'(x) = 1.$$

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

$$\boxed{(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}}$$

Inverse Function Thm.

$f(x) = \sqrt[3]{5x^2+7}$ on $[0, +\infty)$

want $(f^{-1})'(3)$

$$f'(x) = \frac{1}{3}(5x^2+7)^{-2/3} \cdot 10x$$

$$f^{-1}(3) = ?\ 2$$

solve $f(x) = 3$

$f(0) = \sqrt[3]{7}$

$f(1) = \sqrt[3]{12}$

$f(2) = \sqrt[3]{27} = 3$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$$

$$= \frac{1}{f'(2)}$$

$$= \frac{1}{\frac{1}{3}(27)^{-2/3} \cdot 10 \cdot 2}$$

$$= \frac{1}{\frac{1}{3} \cdot \frac{1}{9} \cdot 20} = \frac{1}{\frac{20}{27}} = \frac{27}{20}$$

$f(x) = \sqrt[3]{5x^2+7}$ on $[0, +\infty)$

$f'(x) = \frac{1}{3} (5x^2+7)^{-2/3} \cdot 10x$

want $(f^{-1})'(2)$

$f^{-1}(2) = ?$

solve $f(x) = 2$ $\leftarrow$ approx w/ Newton's method

$f^{-1}(2) \approx .44$

$f(0) = \sqrt[3]{7}$

$(f^{-1})'(2) \approx \dfrac{1}{f'(.44)}$

$f(1) = \sqrt[3]{12}$

$f(2) = \sqrt[3]{27} = 3$