§1 Transcendental Functions

$$\frac{x + \sqrt[3]{x^2} - x^5}{x^3 - x}$$,

trig

sin, cos

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§1.1 Invertible functions

A function is a rule

$x \mapsto f(x)$

$x \mapsto x^2$

$x \mapsto x+3$

Dfn: If f a fn

$g(f(x)) = x$ for every x

then g is an inverse of f.

ex: $f(x) = x \Rightarrow g(x) = x$

$f(x) = 5x + 3 \Rightarrow y = 5x+3$ $\quad g(y) = \frac{y-3}{5}$

$y - 3 = 5x$

$\frac{y-3}{5} = x$

$f(x) = x^3 \Rightarrow g(x) = \sqrt[3]{x}$.

notation: say $g(x) = f^{-1}(x)$.

finding $f^{-1}(y)$ is solving $y = f(x)$ for x.

$f(x) = x^2$

Can I undo w/ $\sqrt{x}$?

no!

if $x^2 = 9$,

then x is 3 or -3.

Can $f^{-1}(x) = \pm\sqrt{x}$

not a fn.

Dfn: f is 1-1 or injective if:

whenever $f(a) = f(b)$

then $a = b$.

ex: • $f(x) = x$

If $f(a) = f(b)$

then $a = b$

• $f(x) = x^3$

if $a^3 = b^3$

then $(a/b)^3 = 1$

so $a/b = 1$ so $a = b$

• $f(x) = \sqrt{x}$ If $\sqrt{a} = \sqrt{b}$

then $(\sqrt{a})^2 = (\sqrt{b})^2$

$a = b$

all 1-1.

$f(x) = x^2$

$f(-3) = 9 = f(3)$

not 1-1.

$f(x) = |x|$

$|-2| = 2 = |2|$

not 1-1

$\sin(x)$

$\sin(0) = 0 = \sin(\pi)$

not 1-1.

$f(x) = 3.$

not 1-1.

Partial inverses

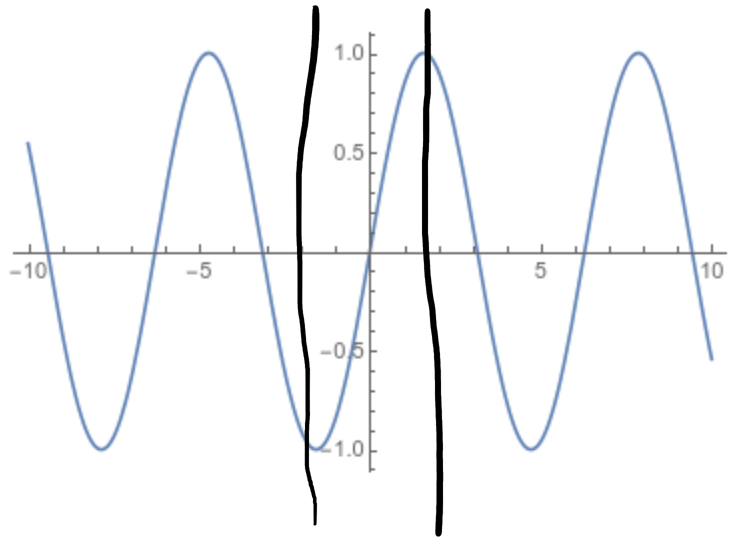
force a fn to be 1-1 by restricting domain.

$f(x) = x^2$ on $[0, +\infty)$

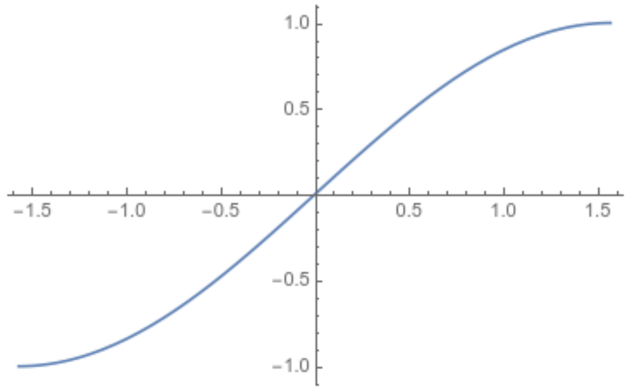
This is 1-1. if $a^2 = b^2$

then $\underbrace{\sqrt{a^2}}_{a} = \sqrt{b^2} = b$

$f^{-1}(x) = \sqrt{x}.$



$\sin(x)$ on $[-\pi/2, \pi/2]$



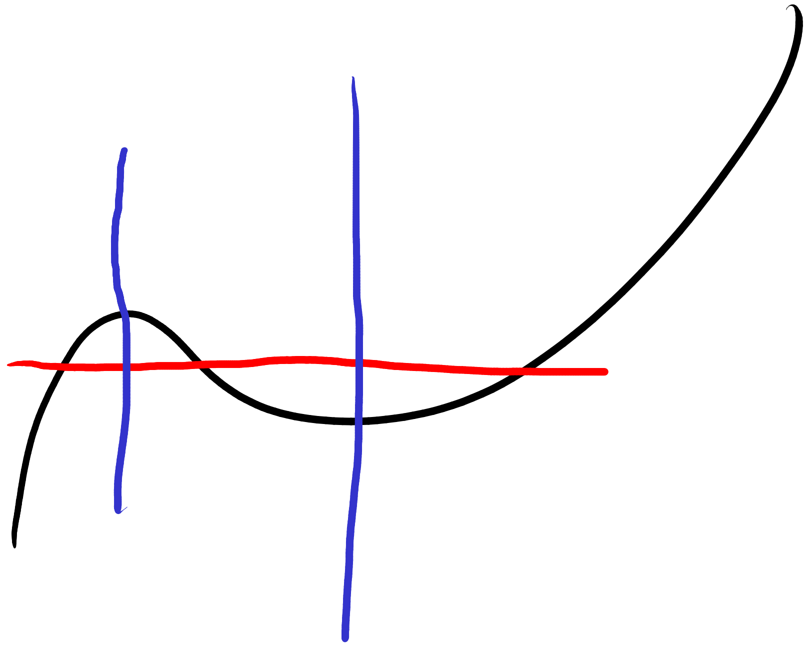
is 1-1.

Horizontal Line Test:

f is 1-1 $\underbrace{\text{if and only if}}_{\text{iff}}$

any horizontal line hits the graph at most once.

a fn has an inverse iff it is 1-1.



Calculus of inverse fns.

if f is 1-1, cts at a,

then f^{-1} is cts at $f(a)$

$x \xmapsto{f} y$ $\qquad \frac{dy}{dx}$

$y \xmapsto{f^{-1}} x$ $\qquad \frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$

$$f^{-1}(f(x)) = x.$$

$$(f^{-1})'(f(x)) \cdot f'(x) = 1.$$

$$(f^{-1})'(f(x)) = \frac{1}{f'(x)}$$

$$\boxed{(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}.}$$

Inverse Function Thm.

$y = f(x)$

$\Downarrow$

$x = f^{-1}(y)$

$f(x) = \sqrt[3]{5x^2+7}$ (on $[0,+\infty)$)

find $(f^{-1})'(3)$.

$f'(x) = \frac{1}{3}(5x^2+7)^{-2/3} \cdot 10x$

$f^{-1}(3) = 2$

$f(0) = \sqrt[3]{7}$

$f(1) = \sqrt[3]{12}$

$f(2) = \sqrt[3]{27} = 3$

$$(f^{-1})'(3) = \frac{1}{f'(f^{-1}(3))}$$

$$= \frac{1}{\frac{1}{3}(5 \cdot 2^2+7)^{-2/3} \cdot 10 \cdot 2}$$

$$= \frac{1}{\frac{1}{3} \cdot \frac{1}{9} \cdot 10 \cdot 2} = \frac{1}{\frac{20}{27}} = \frac{27}{20}.$$

finding $f^{-1}(3)$

is solving $f(x) = 3$.