Math 1232 Spring 2022
Single-Variable Calculus 2 Mastery Quiz 2
Due Tuesday, January 25

This week's mastery quiz has two topics. You should definitely submti an answer to M1. You should only submit an answer to S1 if you did not get a 2 in that topic last week.

Feel free to consult your notes or speak to me privately, but please don't talk about the actual quiz questions with other students in the course or post about it publicly.

Don't worry if you make a minor error, but try to demonstrate that you understand the concepts involved and have mastered the underlying material. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically through Blackboard but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\frac{d}{dx}x^{\ln(x)}$.

Solution: The simplest approach is to use logarithmic differentiation.

$$\begin{aligned} y &= x^{\ln(x)} \\ \ln(y) &= \ln(x)\ln(x) = \ln(x)^2 \\ \frac{y'}{y} &= 2\ln(x)\frac{1}{x} \\ y &= \frac{2\ln(x)}{x}y = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

Alternatively, we could compute

$$\begin{aligned} \frac{d}{dx}x^{\ln(x)} &= \frac{d}{dx}\left(e^{\ln(x)}\right)^{\ln(x)} = \frac{d}{dx}e^{\ln(x)^2} \\ &= e^{\ln(x)^2}\cdot 2\ln(x)\frac{1}{x} = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

(b) $\int \cot(5t)\,dt =$

Solution:

This is a lot like the integral of tangent. We c an think of $\cot(5t)$ as $\cos(5t)/\sin(5t)$. If we take $u = \sin(5t)$ then $du = 5\cos(5t)\,dt$, and we get

$$\begin{aligned} \int \cot(5t)\,dt &= \int \frac{\cos(5t)}{\sin(5t)}\,dt = \int \frac{1}{5}\frac{1}{u}\,du \\ &= \frac{1}{5}\ln|u| + C = \frac{1}{5}\ln|\sin(5t)| + C. \end{aligned}$$

(c) $\int e^x\sqrt{1-3e^x}\,dx$

Solution:

Set $u = 1-3e^x$, so $du = -3e^x\,dx$, and we have

$$\begin{aligned} \int e^x\sqrt{1-3e^x}\,dx &= \int \frac{-1}{3}\sqrt{u}\,du \\ &= \frac{-2}{9}u^{3/2} + C = \frac{-2}{9}(1-3e^x)^{3/2} + C. \end{aligned}$$

S1: Invertible Functions

(a) Find a formula for the inverse of $g(x) = (x-1)^3 + 3$.

Solution:

$$\begin{aligned} y &= (x-1)^3 + 3y - 3 &= (x-1)^3 \\ \sqrt[3]{y-3} &= x - 1 \\ x &= 1 + \sqrt[3]{y-3} \end{aligned}$$

so $g^{-1}(y) = 1 + \sqrt[3]{y-3}$. (You can use whichever variable you like in your formula.)

(b) Let $h(x) = x^5 + x$. Compute $(h^{-1})'(2)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(2) = \frac{1}{h'(h^{-1}(2))}.$$

Guess and check shows that $h(1) = 2$ so $h^{-1}(2) = 1$. And we know that

$$h'(x) = 5x^4 + 1$$

and thus

$$h'(1) = 5 + 1 = 6$$

Thus

$$(h^{-1})'(2) = \frac{1}{6}.$$

(c) Compute $e^{5\ln(3) - 2\ln(4)}$. (Give an exact answer with no decimals.)

Solution:

$$e^{5\ln(3)-2\ln(4)} = (e^{\ln(3)})^5/(e^{\ln(4)})^2 \qquad = \frac{3^5}{4^2} = \frac{243}{16}.$$