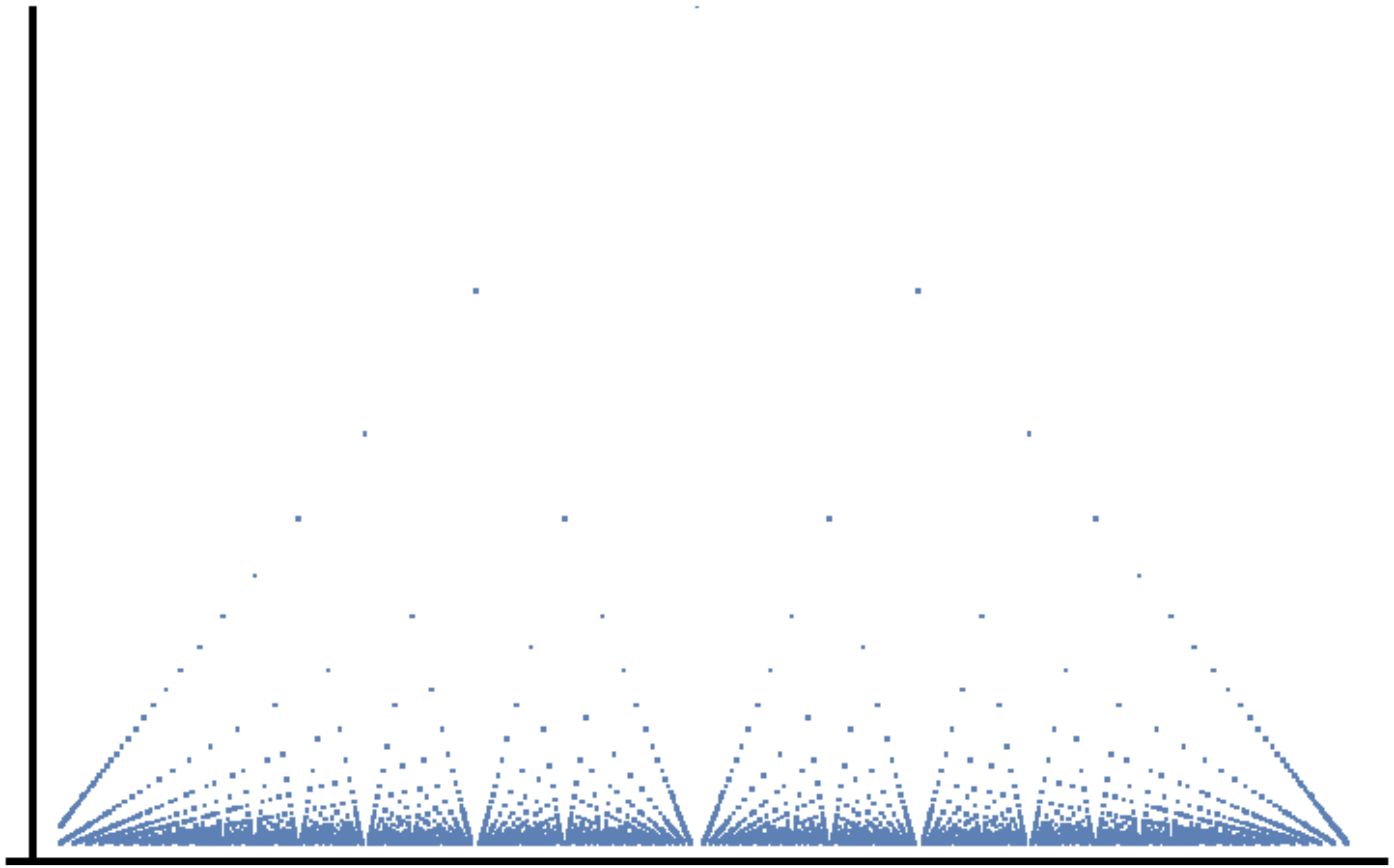
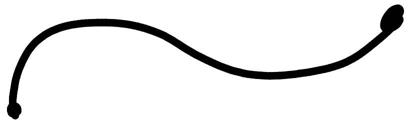
Hi!

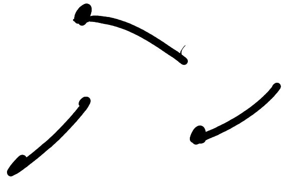


Continuity

f is <u>cts</u> at a if $\lim_{x \to a} f(x) = f(a)$.

Cts if you can draw w/o lifting pencil

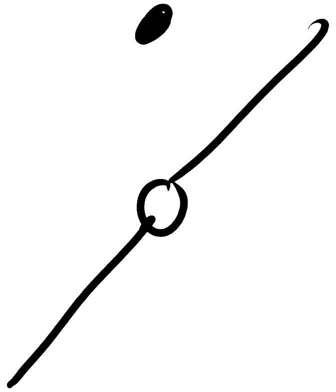




discts

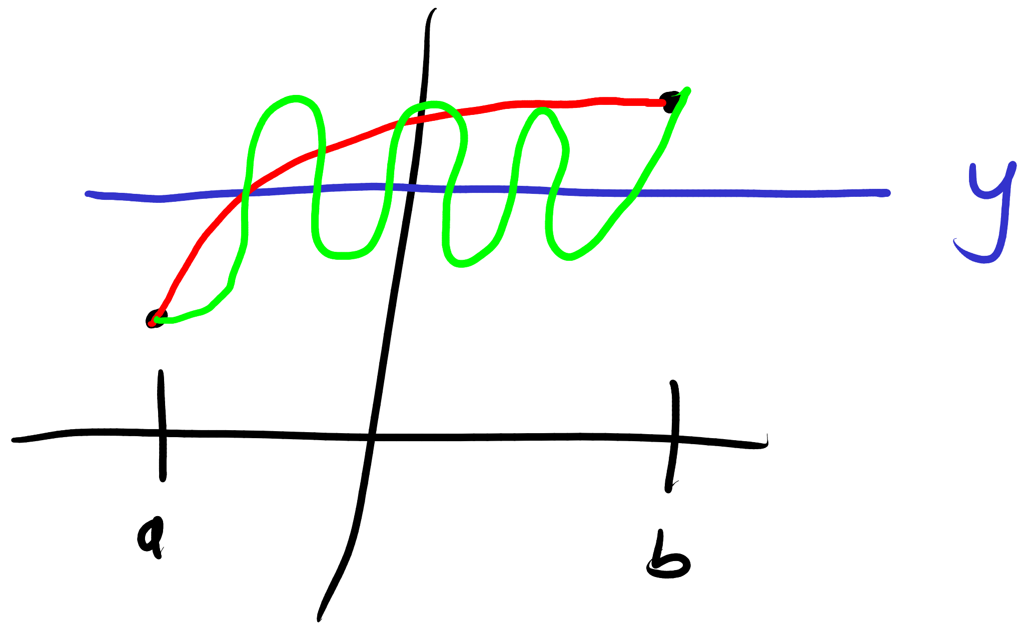
$$\frac{x^2-1}{x-1}$$

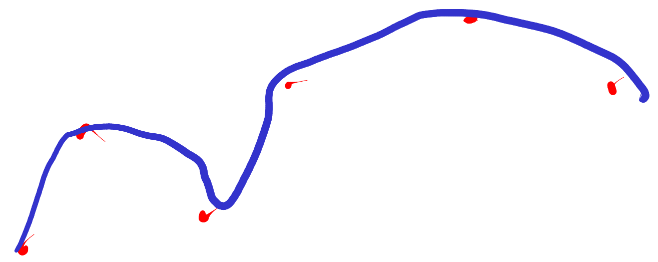
$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

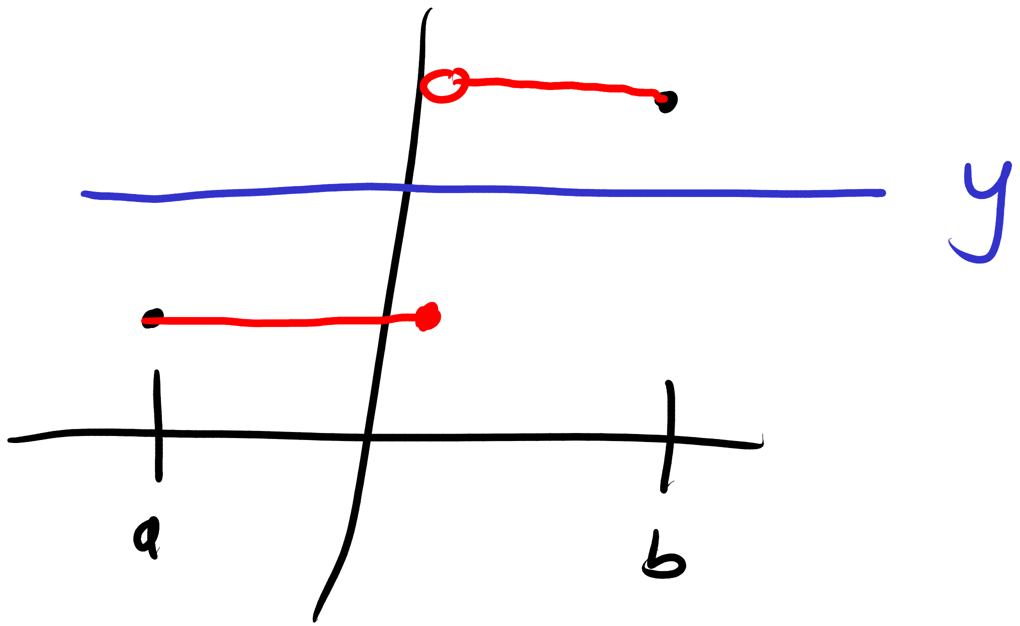


Intermediate Value Theorem

If f is cts on $[a,b]$,
y between $f(a)$ and $f(b)$,
then there is a c in (a,b)
such that $f(c)=y$.







ex! $g(x) = x^3 - x + 1$

Can I solve $g(x) = 1$?

$g(0) = 1$

$g(1) = 1$

$g(4) = y$

Can I solve $g(x) = 4$?

$g(0) = 1$, $g(1) = 1$, $g(2) = 7$, $g(3) = 25$

g is cts on $[1,2]$

$\underbrace{g(1)}_{1} \leq 4 \leq \underbrace{g(2)}_{7}$

So by IVT, there is a c in $(1,2)$ such that $f(c) = 4$.

$g(1.5)$

§ 1.5 Trigonometry

trig fns are cts

$$\lim_{x \to \pi/6} \sin(x) = \sin(\pi/6) = 1/2.$$

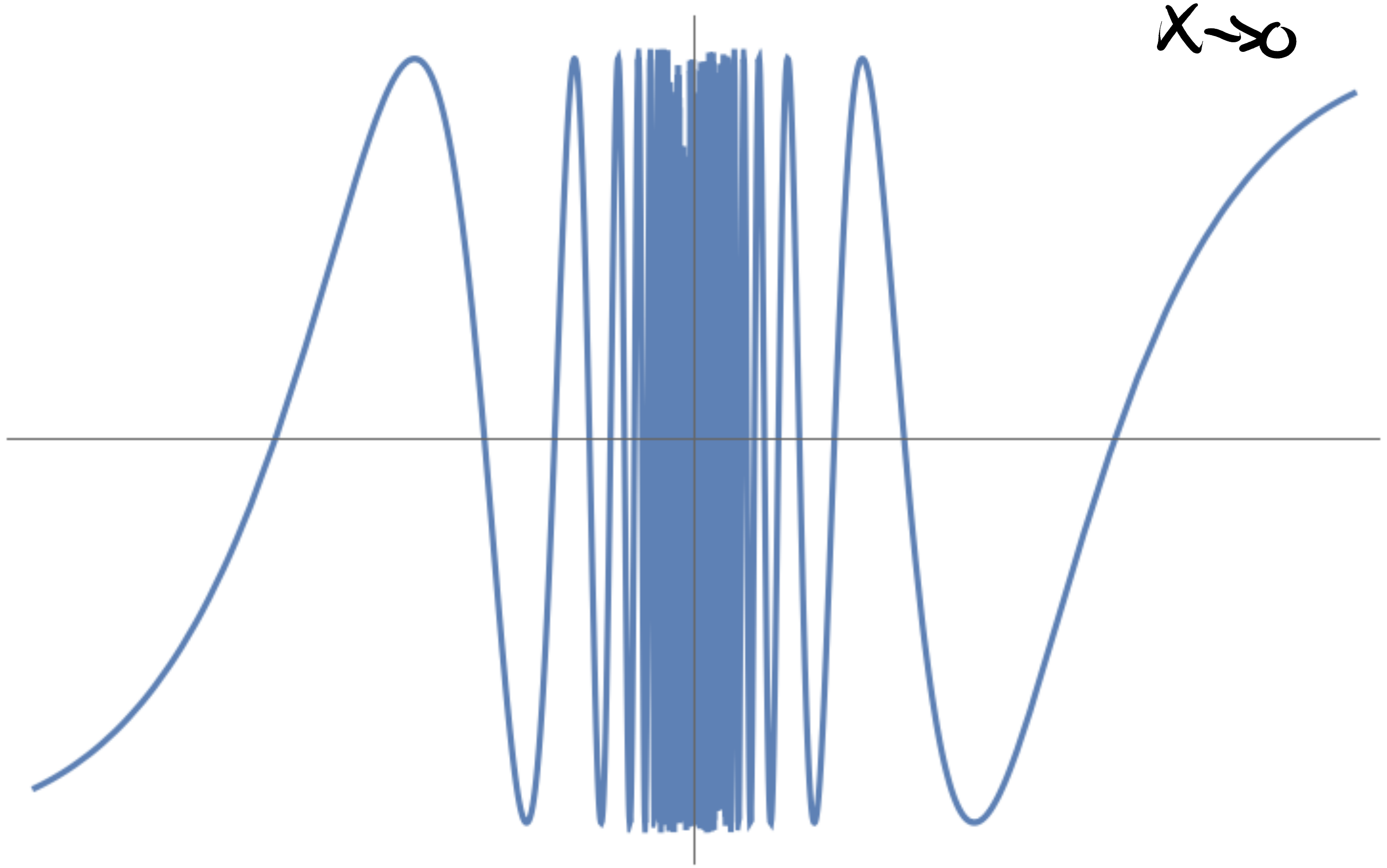
$$\lim_{x \to \pi/4} \tan(x) = \tan(\pi/4) = \frac{\sin(\pi/4)}{\cos(\pi/4)} = 1.$$

Q: $\lim_{x \to 0} \frac{\sin(x)}{x}$?

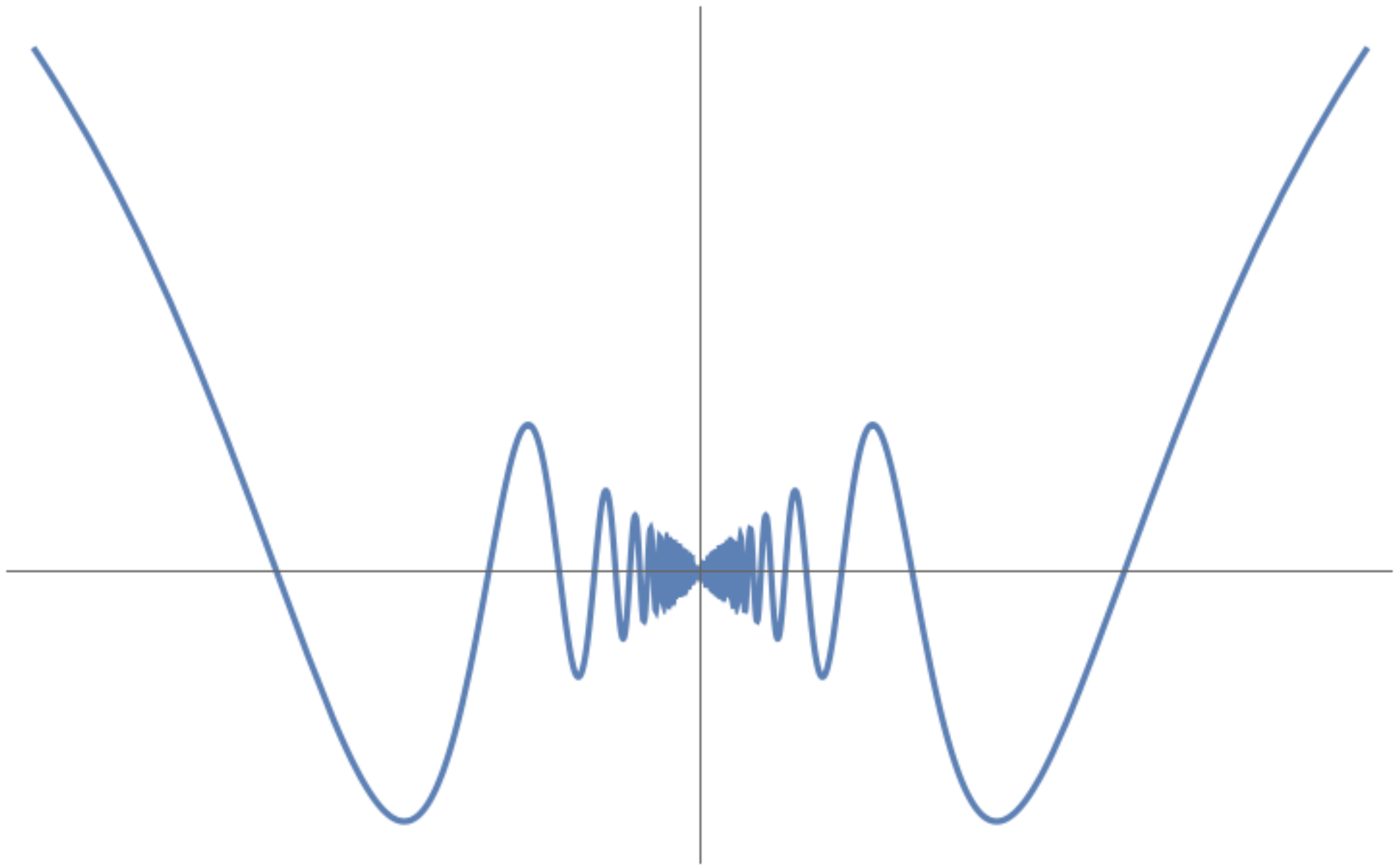
$f(x) = \sin(1/x)$

undefined at 0

$\lim_{x \to 0} \sin(1/x)$ DNE



$g(x) = x \sin(1/x)$



$g(x) = x\sin(1/x)$

$\lim_{x\to 0} \underbrace{x}_{\to 0}\underbrace{\sin(1/x)}_{\text{not too big}}$

$-1 \le \sin(1/x) \le 1$

$-1\cdot x \le x\sin(1/x) \le 1\cdot x$

$-1 \le \sin(1/x) \le 1$

Squeeze Theorem

If $f(x) \le g(x) \le h(x)$ near a

and $\lim_{x\to a} f(x) = \lim_{x\to} h(x)$,

then $\lim_{x\to a} g(x)$ exists and is the same.

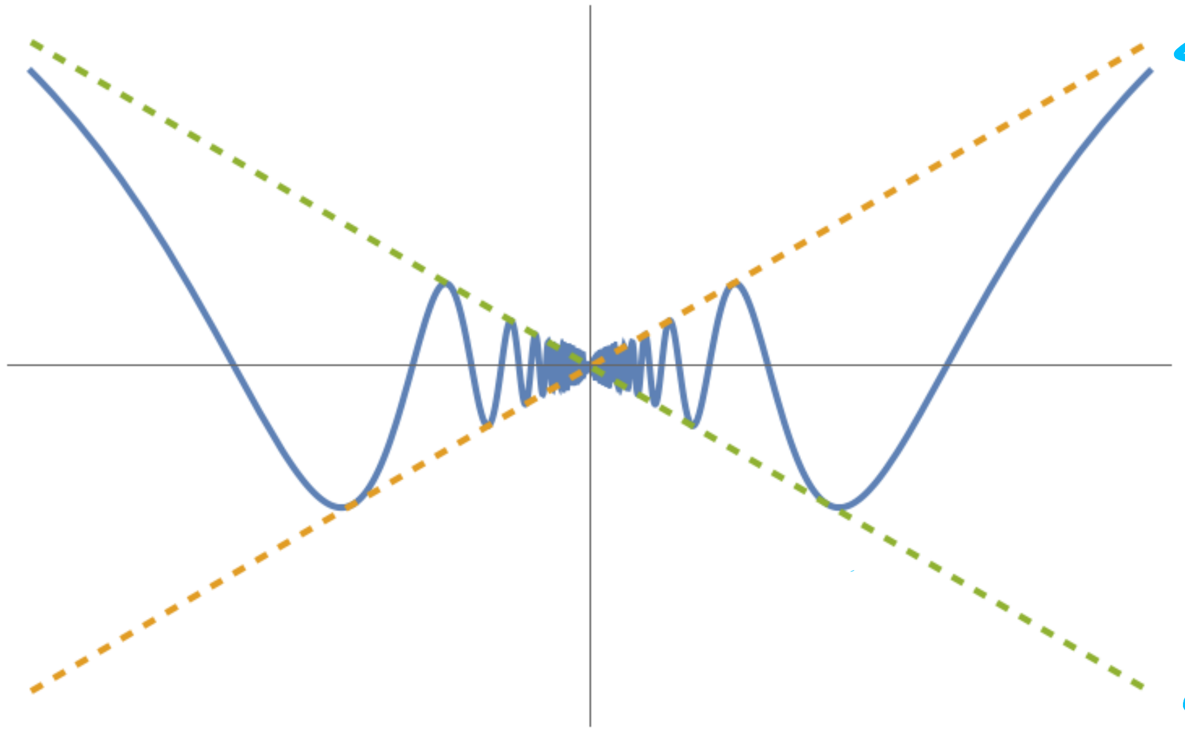
$g(x) = x \sin(1/x)$

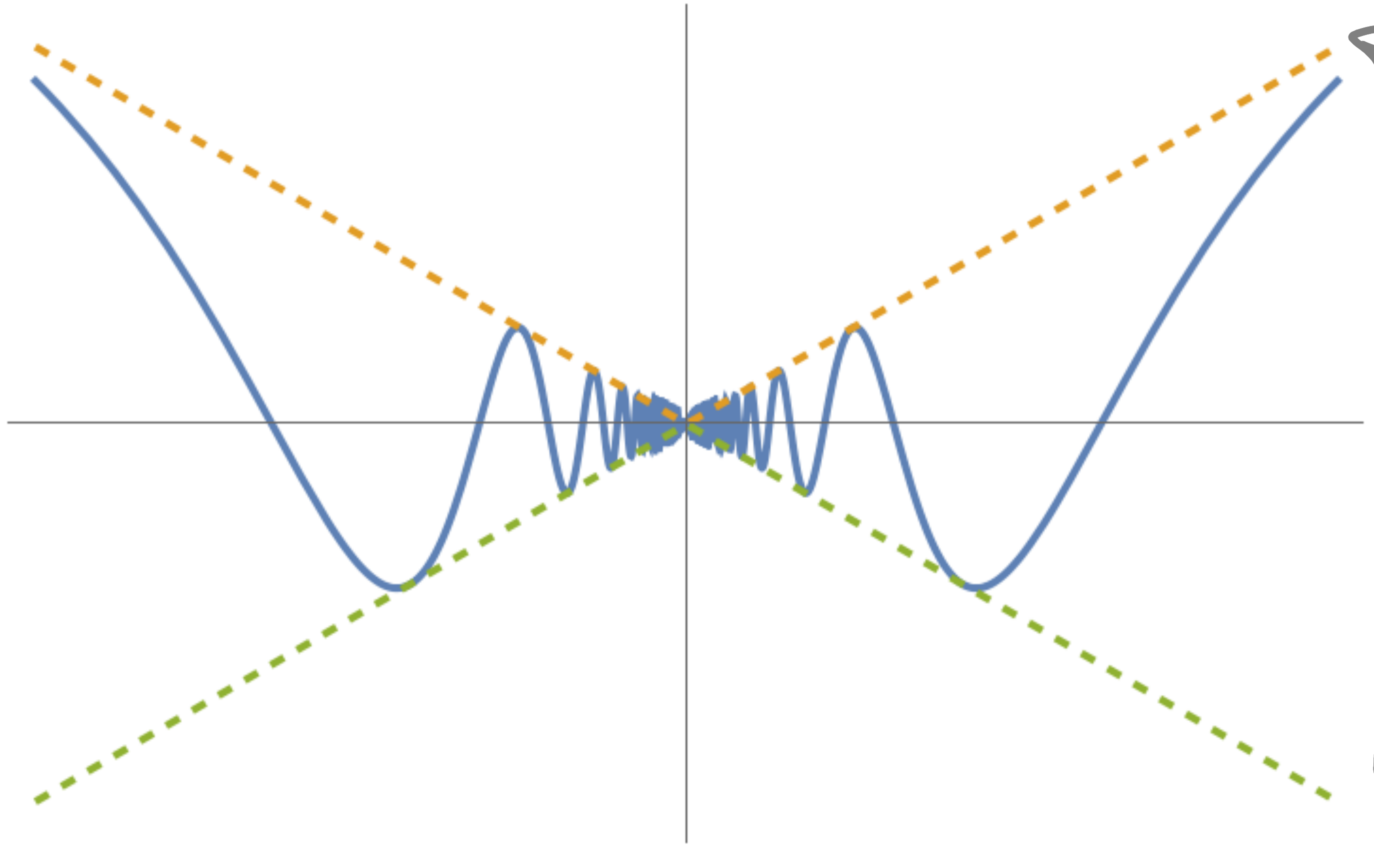
$$\lim_{x \to 0} \underbrace{x}_{0} \underbrace{\sin(1/x)}_{\text{not too big}}$$

$$-1 \le \sin(1/x) \le 1$$

$$\cancel{-1 \cdot x \le x \sin(1/x) \le 1 \cdot x}$$

$$-|x| \le x \sin(1/x) \le |x|$$





$g(x) = x\sin(1/x)$

$$\lim_{x\to 0} \underset{\uparrow\ 0}{x}\ \underbrace{\sin(1/x)}_{\text{not too big}}$$

$$-1 \le \sin(1/x) \le 1$$

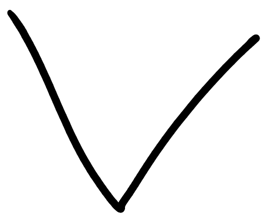
$$\cancel{-1\cdot x \le x\sin(1/x) \le 1\cdot x}$$

$$-|x| \le x\sin(1/x) \le |x|$$

$$\lim_{x\to 0} |x| = 0 \qquad \lim_{x\to 0} -|x| = 0$$

So by squeeze thm,

$$\lim_{x\to 0} x\sin(1/x) = 0.$$



funny, but cts

Small Angle Approximation

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1.$$

$$\cos(\theta) \le \frac{\sin(\theta)}{\theta} \le 1$$

$$\lim_{\theta \to 0} 1 = 1$$

$$\lim_{\theta \to 0} \cos(\theta) = \cos(0) = 1$$

so by squeeze thm,

$$\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1.$$

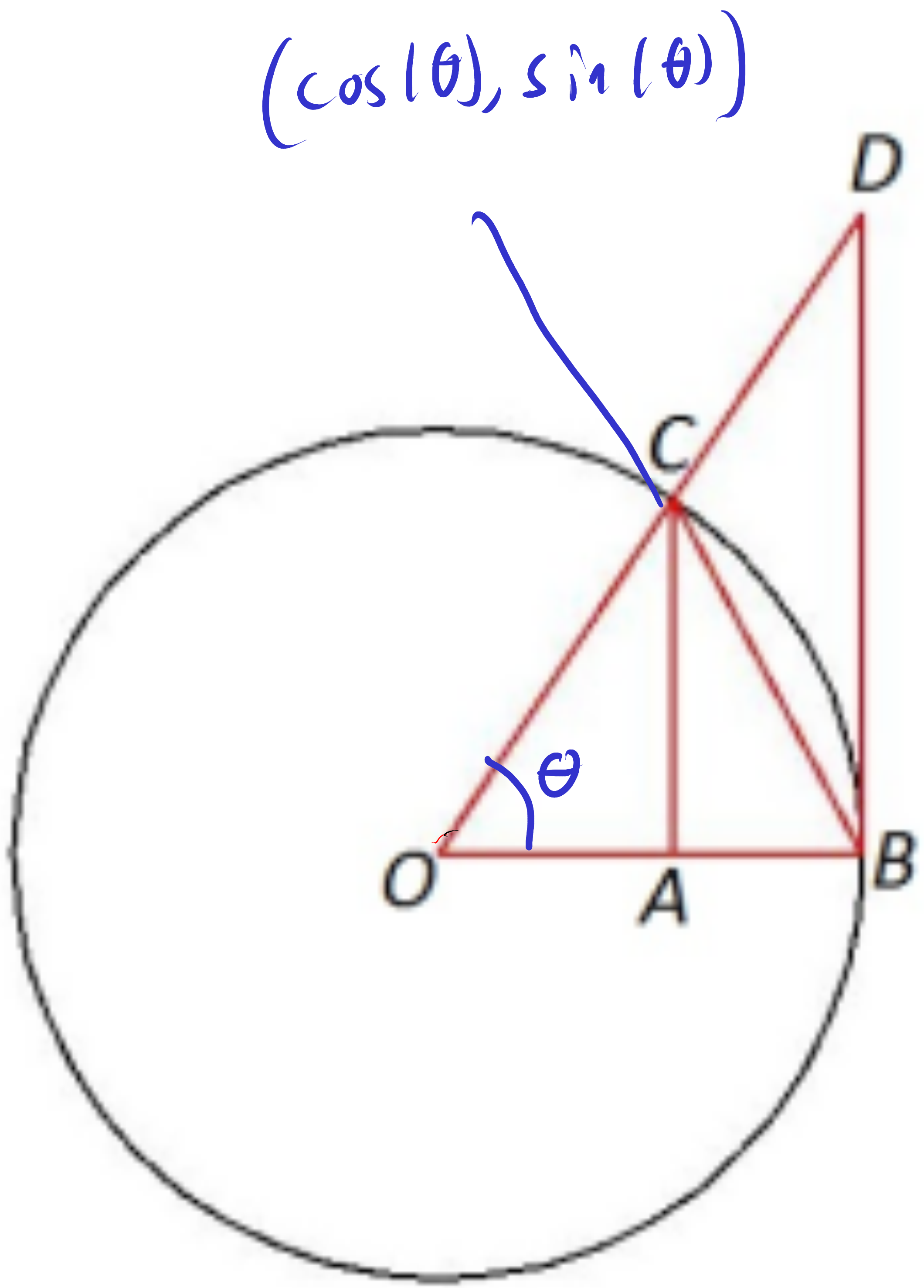
$\sin(\theta) \approx \theta$

Q: $$\lim_{x \to 0} \frac{\sin(2x)}{2x} = 1.$$ $\theta = 2x$

Q: $$\lim_{x \to 0} \frac{\sin(2x)}{x}$$ $\theta = 2x$

$$= \lim_{x \to 0} \cancel{\frac{\sin(2x)}{2x}} \cdot 2 = 2.$$

$$1 \cdot 2$$



$$\triangle OCB = \frac{1}{2}bh = \frac{1}{2}OB \cdot AC$$

$$= \frac{1}{2} \cdot 1 \cdot \sin(\theta) = \frac{\sin(\theta)}{2}$$

$$⌔ OCB = \pi \cdot \frac{\theta}{2\pi} = \frac{\theta}{2}$$

$$\frac{\sin(\theta)}{2} \le \frac{\theta}{2} \Rightarrow \frac{\sin(\theta)}{\theta} \le 1.$$

$$\frac{AC}{OA} = \frac{BD}{OB}$$

$$\frac{\sin\theta}{\cos\theta} = \frac{BD}{1}$$

$$BD = \tan(\theta)$$

$$\triangle ODB = \frac{1}{2}bh = \frac{1}{2} \cdot OB \cdot BD = \frac{1}{2} \cdot 1 \cdot \frac{\sin(\theta)}{\cos(\theta)}$$

$$= \frac{\sin(\theta)}{2\cos(\theta)} \ge \frac{\theta}{2} \Rightarrow \frac{\sin(\theta)}{\theta} \ge \cos(\theta)$$

$$\lim_{x\to 3} \frac{\cancel{\sin(x-3)}}{\cancel{x-3}} = 1.$$

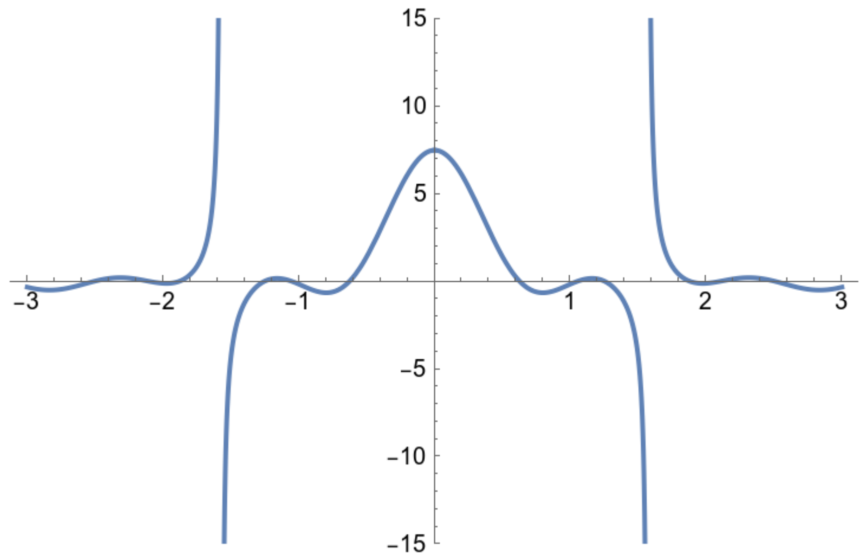
$\theta = x-3$

$$\lim_{x\to 0} \frac{\sin(3x)\,\sin(5x)}{x\,\sin(2x)}$$

$$= \lim_{x\to 0} \frac{\cancel{\frac{\sin(3x)}{3x}}\cdot 3x \cdot \cancel{\frac{\sin(5x)}{5x}}\cdot 5x}{x \quad \cancel{\frac{\sin(2x)}{2x}}\cdot 2x}$$

$\frac{3x}{3x}$ AJF 1

$$= \lim_{x\to 0} \frac{3\cancel{x}\cdot 5\cancel{x}}{\cancel{x}\cdot 2\cancel{x}} \underset{\text{AJF}}{=} \lim_{x\to 0} \frac{3\cdot 5}{2} = \frac{15}{2}$$



$$\frac{\nearrow 3}{\searrow 2} \longrightarrow 3/2$$

$$\frac{\nearrow 3}{\searrow 0} \longrightarrow \infty$$

$$\frac{\nearrow 0}{\searrow 2} \longrightarrow 0$$

$$\frac{\nearrow 0}{\searrow 0} \longrightarrow ??$$

indeterminate form