Math 1231 Practice Midterm Solutions

Instructor: Jay Daigle

Problem 1 (M3)**.** (a) Find and classify all the critical points of $f(x) = (x-5)\sqrt[3]{x^2}$. [Note: this is quite hard but it's good practice.]

(b) The function $g(x) = (x^2 - 3x)\sqrt[3]{x-3}$ has absolute extrema either on $(-4,-1)$ or on $[1,4]$. Pick one of those intervals, explain why g has extrema on that interval, and find the absolute extrema.

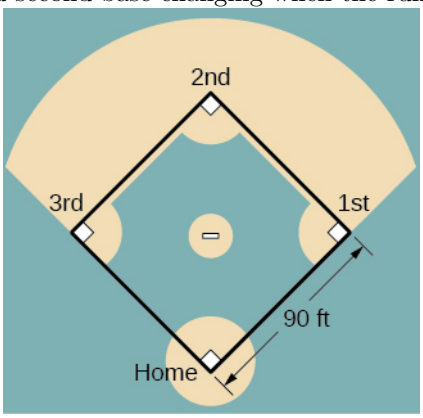
Problem 2 (S4)**.** Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

(a) What are the units of $Q'(p)$? What does it represent physically? What does it mean if $Q'(p)$ is big?

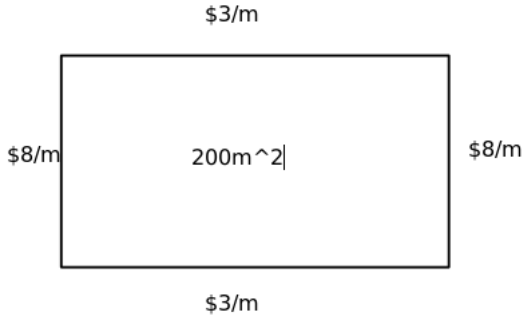
(b) Calculate $Q'(10)$. What does this tell you physically? What physical observation could you make to check your calculation?

Problem 3 (S5)**.** Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

Problem 4 (S6)**.** Consider this baseball diamond, which is a square with 90ft sides. A batter hits the ball and runs from home toward first base at a speed of 22ft/s. At what rate is the distance between the runner and second base changing when the runner has run 30ft?



Problem 5 (S8)**.** We want to build a rectangular fence that will enclose 200m^2. One pair of parallel sides cost \$3/m and the other pair costs \$8/m. What dimensions minimize the cost of the fence? Justify your claim that this is a minimum.



Problem 6 (S7)**.** Let $f(x) = \dfrac{x^3 - 2}{x^4}$. We compute that $f'(x) = \dfrac{8 - x^3}{x^5}$ and $f''(x) = \dfrac{2x^3 - 40}{x^6}$. Sketch a graph of f.

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.