

Math 1231 Practice Midterm

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Problem 1 (M3). (a) Find and classify all the critical points of $f(x) = (x - 5)\sqrt[3]{x^2}$. [Note: this is quite hard but it's good practice.]

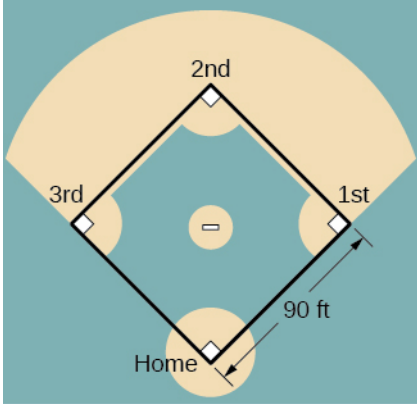
(b) The function $g(x) = (x^2 - 3x)\sqrt[3]{x - 3}$ has absolute extrema either on $(-4, -1)$ or on $[1, 4]$. Pick one of those intervals, explain why g has extrema on that interval, and find the absolute extrema.

Problem 2 (S4). Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

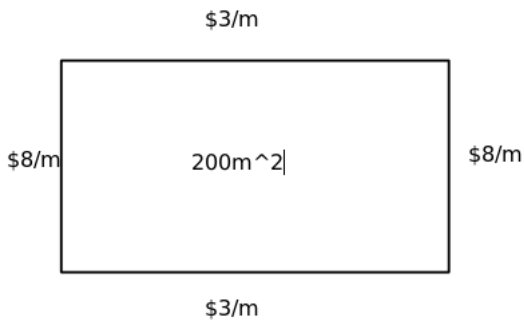
- (a) What are the units of $Q'(p)$? What does it represent physically? What does it mean if $Q'(p)$ is big?
- (b) Calculate $Q'(10)$. What does this tell you physically? What physical observation could you make to check your calculation?

Problem 3 (S5). Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

Problem 4 (S6). Consider this baseball diamond, which is a square with 90ft sides. A batter hits the ball and runs from home toward first base at a speed of 22ft/s. At what rate is the distance between the runner and second base changing when the runner has run 30ft?



Problem 5 (S8). We want to build a rectangular fence that will enclose 200m^2 . One pair of parallel sides cost $\$3/\text{m}$ and the other pair costs $\$8/\text{m}$. What dimensions minimize the cost of the fence? Justify your claim that this is a minimum.



Problem 6 (S7). Let $f(x) = \frac{x^3 - 2}{x^4}$. We compute that $f'(x) = \frac{8 - x^3}{x^5}$ and $f''(x) = \frac{2x^3 - 40}{x^6}$. Sketch a graph of f .

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.