Math 1231: Single-Variable Calculus 1
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Recitation 1

Jay Daigle

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Review Questions

These are all questions on material you should have learned in pre-calculus, but that you might be rusty on. Give them a try and talk to each other! But don't use a calculator; that will get in the way of what you should be learning.

(a) What are the solutions to $x^2 + 5x + 6 = 0$?

(b) What are the solutions to $x^2 + 5x + 5 = 0$?

(c) Compute $(3x + 5)^2$.

(d) Factor $x^3 - 8$.

(e) Compute $\sin(\pi/3)$.

(f) Compute $\tan(5\pi/6)$.

(g) Compute $\sec(-\pi/4)$.

Estimation

The first thing we're going to discuss in class is the idea of *estimation*. Most of the time we compute things exactly: $3 \times 5 = 15$. Other times we estimate: $32 \times 49 \approx 1500$, but this isn't exactly true.

But I want to raise a new question: suppose we want to get our output really close to some target. So we want, e.g., to get 1500 ± 25. We can see that 32×49 is close-ish to 1500, but not that close!

Problem 1. A cylindrical water tank has a base with an area of six square inches. We want to fill it with water ten inches deep, to the nearest inch.

(a) What is the exact volume of water would we ideally want to pour in?

(b) What is the least height that would satisfy our requirements? What is the least volume of water we can pour into the tank and still satisfy our height requirements?

(c) What is the greatest height that would satisfy our requirements? What is the greatest volume of water we can pour into the tank and satisfy our height requirements?

(d) How much error can we have in the volume? That is, how far away from our ideal volume in part (a) can we get and still be safe?

Problem 2. We know that $\sqrt{4} = 2$. Suppose we want to have $1 \leq \sqrt{x} \leq 3$. We might describe this as wanting $\sqrt{x} = 2 \pm 1$. If we're being fancy, we use the Greek letter ε and say that we want an error tolerance of $\varepsilon = 1$.

(a) What is the largest input that keeps the output within ε of 2?

(b) What is the smallest input that keeps the output within ε of 2?

(c) Our ideal input is 4. How much can we miss our ideal input by? (When we're being fancy, we call this number by the Greek letter δ).

Now instead let's take $\varepsilon = .5$.

(d) What is the largest input that keeps the output within ε of 2?

(e) What is the smallest input that keeps the output within ε of 2?

(f) What does that make δ?

In class, we looked at the following question: Suppose we want to make a square platform that's 16 square meters, plus or minus 1. How long do the sides need to be?

Clearly, our sides need to be between $\sqrt{15}$ and $\sqrt{17}$ but that doesn't tell us anything useful. So instead we made the following argument: We can use an absolute value to describe the way we think about errors. in particular, what we want here is

$$|s^2 - 16| < \varepsilon = 1, \tag{1}$$

and factoring the left hand side gives $|s-4| \cdot |s+4| < 1$. We can't solve this exactly, but we can make the following lazy decision: We know s should be *approximately* 4. It might be a little bigger, so $s+4$ might be bigger than 8, but it's certainly less than 9, or 10. Then we just need to solve

$$|s-4| \cdot |s+4| < 10|s-4| < 1 \tag{2}$$

$$|s-4| < .1 \tag{3}$$

$$-.1 < s-4 < .1 \tag{4}$$

$$3.9 < s-4 < 4.1. \tag{5}$$

Thus $\delta = .1$ and s should be $4 \pm .1$.

This is a tricky argument! But I want you to try to think through it now.

Problem 3. Let's suppose instead we want to make a square platform with area 25 square meters, plus or minus 1.

1. Write down the analogue of inequality (1) for this new problem. Can you explain in words what this inequality says about your error?

2. We can factor the left-hand side of this inequality into two factors. If our input is close to 5, one of these terms will be small, and the other will be large. Which one will be large, and about how large will that be?

3. This should let you write down an inequality like the one in (2). What is it?

4. Figure out δ such that $s = 5 \pm \delta$ will keep us in our error bounds.

5. Check your answer: square $5+\delta$ and $5-\delta$ and see whether the answers fall within your error margin.

6. Could you use a larger δ than the one you found in part (4)?