Math 1231 Fall 2022
Single-Variable Calculus I Section 13
Mastery Quiz 2
Due Thursday, September 7

This week's mastery quiz has two topics. Everyone should submit work for M1. If you got a 2/2 on S1 last week, you don't need to submit that topic again, but if you got a 0 or 1 you should try again.

Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Thursday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Secondary Topic 1: Estimation
- Major Topic 1: Computing Limits

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x \to -2} \frac{x+2}{\sqrt{x+6}-2} =$

Solution:

$$\lim_{x \to -2} \frac{x+2}{\sqrt{x+6}-2} = \lim_{x \to -2} \frac{(x+2)(\sqrt{x+6}+2)}{x+6-4}$$
$$= \lim_{x \to -2} \frac{\sqrt{x+6}+2}{1} = 4.$$

(b) $\lim_{x \to 2} \frac{x^2+x-5}{3-x} =$

Solution:

$$\lim_{x \to 2} \frac{x^2+x-5}{3-x} = \frac{1}{1} = 1.$$

(c) $\lim_{x \to 1} \frac{1}{x-1} - \frac{1}{x^2-x} =$

Solution:

$$\lim_{x \to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x \to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x \to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x \to 1} \frac{1}{x} = 1.$$

Secondary Topic 1: Estimation

(a) We want to build a ramp that's eight times long as it is tall, and we want it to reach a height of 10 meters. Find a formula for δ in terms of ϵ, so that if the error in the *length* is less than δ then the error in the height is less than ϵ. Make sure your formula gives the **largest** δ **possible**, and justify your answer.

Solution: The height is $L/8$, so our output error is $|L/8 - 10| = \frac{1}{8}|L-80|$, which we want to be less than ε. So we get

$$|L/8 - 10| = \frac{1}{8}|L-80| < \varepsilon$$
$$|L-80| < 8\varepsilon.$$

So if we take $\delta = 8\varepsilon$, then whenever the error in the length of our ramp $|L-80|$ is less than δ, then the error in our height should be less than ε.

(b) Suppose $f(x) = \sqrt{x+1}$, and we want an output of approximately 3. What input a should we aim for? Find a δ so that if our input is $a \pm \delta$ then our output will be $3 \pm .5$. Explain how you found this δ and why it should give us what we want.

Solution: We want an input of about $a = 8$. By solving the equation we can see that if

$$\begin{aligned} f(x) &= 2.5 \\ x + 1 &= 2.5^2 = 6.25 \\ x &= 5.25 \end{aligned} \qquad\qquad \begin{aligned} f(x) &= 3.5 \\ x + 1 &= 3.5^2 = 12.25 \\ x &= 11.25 \end{aligned}$$

so we want x in $(5.25, 11.25)$. This ranges from $8 - 2.75$ to $8 + 3.25$, so we take $\delta = 2.75$ as the smaller of these two distances.