

Math 1231-13: Single-Variable Calculus 1
George Washington University Fall 2023
Recitation 2

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Today we want to practice the way we actually compute limits.

Problem 1 (Warmup). Let $f(x) = \frac{x^2 + \sin(x) + 3}{x^2 - x - 2}$.

- (a) Where is f continuous? Where is it discontinuous?
- (b) What is $\lim_{x \rightarrow 0} f(x)$?

Problem 2. Let $f(x) = \frac{x-1}{x^2-1}$.

- (a) What is $f(2)$? Is f continuous at 2?
- (b) What is $\lim_{x \rightarrow 2} f(x)$?
- (c) What is $f(1)$? Is f continuous at 1?
- (d) What function can we find that's almost the same as f , but defined and continuous at 1? (Is this function the same as f ?)
- (e) What is $\lim_{x \rightarrow 1} f(x)$?

Problem 3. Let $g(x) = \frac{(x+1)^2 - 1}{x+2}$.

- (a) Is g continuous where it's defined? Where is it undefined?
- (b) Can you find a function that's almost identical to g but continuous everywhere?
- (c) What is $\lim_{x \rightarrow -2} g(x)$?

Problem 4. Let $h(x) = \frac{x-1}{\sqrt{5-x}-2}$.

- (a) Is this function continuous where it's defined? Where is it undefined?
- (b) We can factor an $x - 1$ out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the $x - 1$ appear. What tricks do we have that might work?
- (c) What is $\lim_{x \rightarrow 1} h(x)$?

Problem 5. We want to compute $\lim_{x \rightarrow 3} \frac{\sin(x^2-9)}{x-3}$.

- (a) What rule do we know we need to invoke here?
- (b) What θ are we going to need to pick for this to work out, and why?
- (c) Do algebra so that you can invoke the small angle approximation. What is the limit? (Are you using the AIF property?)
- (d) Go back to the beginning, and see what our heuristic idea that $\sin(\theta) \approx \theta$ would have told you. Does that match with what you got?