# Math 1231 Practice Midterm Solutions

Instructor: Jay Daigle

- You will have 75 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 4 problems, one on each mastery topic we've covered. The exam has 3 pages total.
- Each part of each topic is worth ten points. The whole test is scored out of 90 points.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

  When in doubt, show more work and write complete sentences.
- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

**Name:**

**Recitation Section:**

| | a | b | c | d |
|---|---|---|---|---|
| M1 | | | | |
| M2 | | | | ■ |
| S1 | | S2 | | ■ |
| $\sum$ | | | /90 | ■ |

**Problem 1** (M1)**.** Compute the following limits if they exist. Make sure you **justify your answers** using the tools we have developed in class.

(a) $\lim_{x\to 2} \frac{1}{x-2} - \frac{3}{(x+1)(x-2)} =$

**Solution:**

$$\begin{aligned}\lim_{x\to 2} \frac{1}{x-2} - \frac{3}{(x+1)(x-2)} &= \lim_{x\to 2} \frac{x+1}{(x+1)(x-2)} - \frac{3}{(x+1)(x-2)} \\ &= \lim_{x\to 2} \frac{x-2}{(x+1)(x-2)} = \lim_{x\to 2} \frac{1}{x+1} = \frac{1}{3}.\end{aligned}$$

(b) $\lim_{x\to+\infty} \frac{\sqrt{5x^4+x+1}}{x^2+3x+4} =$

**Solution:**

$$\lim_{x\to+\infty} \frac{\sqrt{5x^4+x+1}}{x^2+3x+4} = \lim_{x\to+\infty} \frac{\sqrt{5+1/x^3+1/x^4}}{1+3/x+4/x^2} = \frac{\sqrt{5}}{1} = \sqrt{5}.$$

(c) $\lim_{x\to 0} \frac{\sin(3x)\tan(2x)}{5x^2} =$

**Solution:**

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(3x)\tan(2x)}{5x^2} &= \lim_{x\to 0} \frac{\frac{\sin(3x)}{3x}\cdot 3x \cdot \frac{\sin(2x)}{2x}\frac{2x}{\cos(2x)}}{5x^2} \\ &= \lim_{x\to 0} \frac{3x\cdot 2x}{5x^2\cos(2x)} \\ &= \lim_{x\to 0} \frac{6}{5\cos(2x)} = \frac{6}{5}.\end{aligned}$$

(d) $\lim_{x\to -3} \frac{\sqrt{x+4}-1}{(x+3)^2} =$

**Solution:**

$$\begin{aligned}\lim_{x\to -3} \frac{\sqrt{x+4}-1}{(x+3)^2} &= \lim_{x\to -3} \frac{x+4-1}{(x+3)^2(\sqrt{x+4}+1)} \\ &= \lim_{x\to -3} \frac{x+3}{(x+3)^2(\sqrt{x+4}+1)} \\ &= \lim_{x\to -3} \frac{1^{\nearrow 1}}{(x+3)(\sqrt{x+4}+1)_{\searrow 0}} = \pm\infty.\end{aligned}$$

**Problem 2** (M2)**.** (a) Explicitly justifying each step and naming each derivative rule you use, compute $\frac{d}{dx}3x\cos(x) + \frac{4}{x}$.

**Solution:**

$$\begin{aligned}
\frac{d}{dx}3x\cos(x)+\frac{4}{x} &= \Big(3x\cos(x)\Big)'+\left(\frac{4}{x}\right)' && \text{Sum Rule}\\
&= 3\Big(x\cos(x)\Big)'+4\Big(x^{-1}\Big)' && \text{Scalar Products}\\
&= 3\Big((x)'\cos(x)+x(\cos(x))'\Big)+4\Big(x^{-1}\Big)' && \text{Product Rule}\\
&= 3\Big(\cos(x)+x(\cos(x))'\Big)+4\Big(x^{-1}\Big)' && \text{Identity}\\
&= 3\Big(\cos(x)+x(\cos(x))'\Big)+4\left(-x^{-2}\right) && \text{Power Rule}\\
&= 3\left(\cos(x)+x(-\sin(x))\right)+4\left(-x^{-2}\right) && \text{Derivative of Cosine.}
\end{aligned}$$

(b) Using any methods we've developed in class, compute the derivative of $g(x)=\tan^5(x^3+x)$.

**Solution:**

$$g'(x)=5\tan^4(x^3+x)\sec^2(x^3+x)(3x^2+1).$$

(c) Using any methods we've developed in class, compute the derivative of $h(x)=\dfrac{x^3+\sec(x)}{\sqrt[3]{x}+5}$.

**Solution:**

$$h'(x)=\frac{(3x^2+\sec(x)\tan(x))(\sqrt[3]{x}+5)-\frac{1}{3}x^{-2/3}(x^3+\sec(x))}{(\sqrt[3]{x}+5)^2}.$$

**Problem 3** (S1)**.**
Suppose $f(x)=5x+4$, and we want an output of approximately 14. What input $a$ should we aim for? Find a formula for $\delta$ in terms of $\varepsilon$ so that if our input is $a\pm\delta$ then our output will be $14\pm\varepsilon$. Justify your answer.

**Solution:** We want an input of about $a=2$. Our output error will be $|5x+4-14|=|5x-10|=5|x-2|$. We want this to be less than $\varepsilon$, so we need

$$\begin{aligned}
5|x-2| &< \varepsilon\\
|x-2| &< \varepsilon/5
\end{aligned}$$

so we can take $\delta=\varepsilon/5$.

**Problem 4** (S2)**. Directly from the definition of derivative,** compute the derivative of $f(x)=(x-3)^2$ at $a=5$.

**Solution:**

$$\begin{aligned}
f'(5) &= \lim_{h\to 0}\frac{f(5+h)-f(5)}{h}\\
&= \lim_{h\to 0}\frac{(5+h-3)^2-4}{h}\\
&= \lim_{h\to 0}\frac{(2+h)^2-4}{h}\\
&= \lim_{h\to 0}\frac{4+4h+h^2-4}{h}\\
&= \lim_{h\to 0}\frac{4h+h^2}{h}=\lim_{h\to 0}4+h=4.
\end{aligned}$$