

Math 1231 Practice Midterm

Instructor: Jay Daigle

- You will have 75 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator. You may leave answers unsimplified, except you should compute trigonometric functions as far as possible.
- The exam has 5 problems, one on each mastery topic we've covered. The exam has 5 pages total.
- M3 and M4 are each worth 20 points. S3 and S4 are each worth 10 points. S5 is worth 15 points. The whole test is scored out of 75 points.
- Read the questions carefully and make sure to answer the actual question asked. Make sure to justify your answers—math is largely about clear communication and argument, so an unjustified answer is much like no answer at all.

When in doubt, show more work and write complete sentences.

- If you need more paper to show work, I have extra at the front of the room.
- Good luck!

Problem 1 (M3).

(a) Find a tangent line to the curve given by $x^4 - 2x^2y^2 + y^4 = 16$ at the point $(\sqrt{5}, 1)$.

(b) Give equation for the linear approximation of the function $f(x) = x \sin(x)$ near the point $a = \pi/2$.
Use it to estimate $f(1.5)$.

Problem 2 (M4).

(a) Find and classify all the critical points of $f(x) = (x - 5)\sqrt[3]{x^2}$.

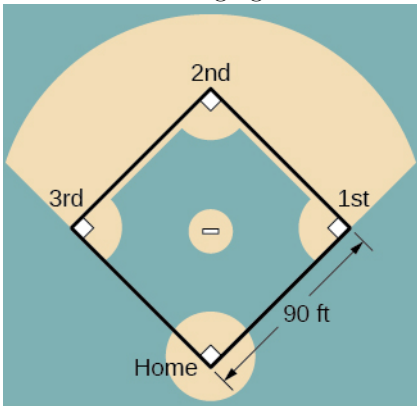
(b) Find the absolute extrema of $g(x) = (x^2 - 3x)\sqrt[3]{x - 3}$ on $[1, 4]$, and justify your claim that these are the absolute extrema.

Problem 3 (S3). Suppose that $Q(p) = 3p^2 + 10p - 100$ is the number of widgets you can buy at a price of p dollars.

(a) What are the units of $Q'(p)$? What does it represent physically? What does it mean if $Q'(p)$ is big?

(b) Calculate $Q'(10)$. What does this tell you physically? What physical observation could you make to check your calculation?

Problem 4 (S4). Consider this baseball diamond, which is a square with 90ft sides. A batter hits the ball and runs from home toward first base at a speed of 22ft/s. At what rate is the distance between the runner and second base changing when the runner has run 30ft?



Problem 5 (S5). Let $f(x) = \frac{x^3 - 2}{x^4}$. We compute that $f'(x) = \frac{8 - x^3}{x^5}$ and $f''(x) = \frac{2x^3 - 40}{x^6}$. Sketch a graph of f .

Your answer should discuss the domain, asymptotes, roots, limits at infinity, critical points and values, intervals of increase and decrease, and concavity.