Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 1
Due Tuesday, January 24

This week's mastery quiz has one topic. Please submit your best attempt at answering the questions on that topic, and try to demonstrate your mastery of the underlying material. (And don't worry if your answers aren't completely solid; this is a quiz but it's also a learning experience, and you'll get another try next week.)

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Secondary Topic 1: Estimation

Name:

Recitation Section:

Secondary Topic 1: Estimation

(a) Suppose $f(x) = x^2 + 4x$, and we want an output of approximately 12. What input a should we aim for, if we want a positive input? Find a δ so that if our input is $a \pm \delta$ then our output will be 12 ± 1. Justify your answer.

Solution: We want an input of about $a = 2$. Our output error will be $|x^2+4x-12| = |x+6| \cdot |x-2|$. We know that $x + 6 \approx 8 < 9$, so we have

$$|x^2 + 4x - 12| = |x + 6| \cdot |x - 2| < 9|x - 2| < 1$$

so we need $|x - 2| < 1/9$. So we can take $\delta = 1/9$.

(b) We want to amplify an electrical signal. Our amplifier will multiply the voltage by a factor of six, and we want an output signal of $24 \pm \varepsilon$ volts. Find a formula for δ in terms of ε, so that if the input error is less than δ then the error in the output is less than ε. Make sure your formula gives the **largest** δ **possible**, and justify your answer.

Solution: Our output error is $|6x - 24| = 6|x - 4|$, and we want this to be less than ε. So we get

$$\begin{aligned} |6x - 24| = 6|x - 4| &< \varepsilon \\ |x - 4| &< \varepsilon/6. \end{aligned}$$

So if we take $\delta = \varepsilon/6$, then whenever the error in our input voltage $|x - 4| < \delta$ then the error in our output voltage should be less than ε.