Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 10
Due Tuesday, April 4

This week's mastery quiz has three topics. Everyone should submit work on S6. If you already have a 4/4 on M4, you should not submit it. But if Blackboard doesn't say you're at a 4/4, then you should submit again for another try. If you already have a 2/2 on S5, you don't need to submit it.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 4: Optimization
- Secondary Topic 5: Curve Sketching
- Secondary Topic 6: Physical Optimization

Name:

Recitation Section:

M4: Extrema and Optimization

(a) Classify all the critical points and relative extrema of $h(x) = x^3/(x+1)$. (For each critical point, tell me whether it is a relative maximum, a relative minimum, or neither.)

(b) Find the absolute extrema of $f(x) = 3x^4 - 20x^3 + 24x^2 + 7$ on $[0, 5]$.

S5: Curve Sketching

Sketch the graph of $f(x) = x^5 - 5x^4 + 5x^3 = x^3(x^2 - 5x + 5)$. We have $f'(x) = 5x^2(x-3)(x-1)$ and $f''(x) = 10x(2x^2 - 6x + 3)$.

You should discuss the domain, limits at infinity, critical points, intervals of increase and decrease, concavity, and possible points of inflection.

S6: Physical Optimization

We wish to build a rectangular pen with two parallel internal partitions, using 1000 feet of fencing. We want to maximize the total area of the pen.

(a) What is your objective function, and why?

(b) What constraint equation(s) can you use?

(c) What dimensions maximize the total area of the pen?