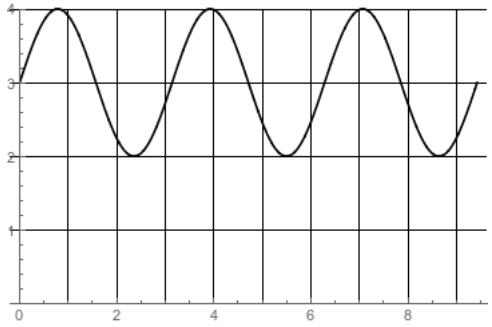
Math 1231: Single-Variable Calculus 1
George Washington University Spring 2023
Recitation 11

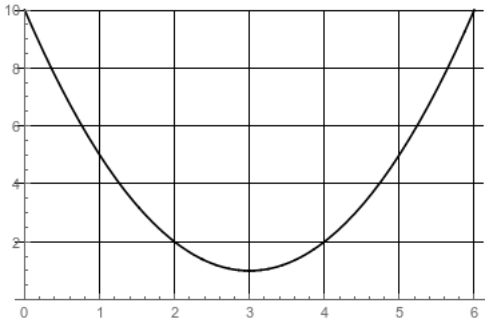
Jay Daigle

April 7, 2023

Problem 1. For the following curves, find an upper bound and a lower bound for the area under the curve, and then give your best estimate for the actual area.



(between 0 and 9; ignore the trailing bit off the right edge)



(between 0 and 6; ignore the trailing bit off the right edge)

Problem 2. Consider the function $f(x) = \sqrt{1-x^2}$ between $x = 0$ and $x = 1$.

(a) Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?

(b) Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?

(c) Find an upper bound using four rectangles.

(d) Find a lower bound using four rectangles.

(e) Can you guess what the area under the curve is exactly? (Hint: what does the graph look like?)

Problem 3. Consider the function $g(x) = x^3$ between $x = 0$ and $x = 1$.

(a) Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?

(b) Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?

(c) Find an upper bound using four rectangles.

(d) Find a lower bound using four rectangles.

(e) Find a formula using right endpoints to estimate the area using n rectangles, in summation form.

(f) Use your summation rules to get a closed-form formula, with no summation signs in it.

(g) Take a limit to find the exact area under this curve. (Use your summation rules!)

Problem 4. Let $h(x) = x^2 + 3x - 2$ between $x = -2$ and $x = 2$.

(a) Write down a summation formula for R_n.

(b) Use summation rules to get a closed-form formula.

(c) Take a limit to compute $\int_{-2}^{2} x^2 + 3x - 2\,dx$.

(d) What does this refer to geometrically? Does that make sense?