

Math 1231: Single-Variable Calculus 1
George Washington University Spring 2023
Recitation 11

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Problem 1. Suppose we know that $\int_2^4 f(x) dx = 3$, $\int_4^6 f(x) dx = 5$, and $\int_2^6 g(x) dx = -2$. Compute the following integrals, justifying your answers:

(a) $\int_2^4 3f(x) dx$?

(b) $\int_2^6 f(x) - g(x) dx$?

(c) $\int_6^4 f(x) - 3 dx$?

Problem 2. (a) Use the Fundamental Theorem of Calculus to compute $\frac{d}{dx} \int_2^x \sqrt{t^5 - t} dt$.

(b) Compute $\frac{d}{dx} \int_x^5 s^5 + \cos(s^2) ds$. What rule did you have to use here other than the FTC?

(c) Compute $\frac{d}{dx} \int_{-3}^{x^2} \sqrt{t^3 + 1} dt$. What rule did you have to use here other than the FTC?

Problem 3. We want to find $\frac{d}{dx} \int_{3x}^{x^3} \sqrt[3]{x+1} dx$. Unfortunately we can't apply the Fundamental Theorem of Calculus directly.

(a) This integral has variables in both the upper and lower bounds. Can you split it into multiple integrals, each of which has only one variable in a bound?

(b) To use the FTC we need the variable as the *upper* bound of each integral. How can we do that?

(c) Now for each integral you have set up, carefully take the derivative, paying attention to the chain rule.

(d) Combine this work to answer the original question.

Problem 4. Compute the following integrals:

(a) $\int x(x + 1) dx$

(b) $\int x\sqrt{x} dx$

(c) $\int 5 \csc(x) \cot(x) dx$

(d) $\int (x^4 - x)(x^2 + x + 1) dx.$