Math 1231: Single-Variable Calculus 1
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Recitation 11

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Problem 1. Suppose we know that $\int_2^4 f(x)\,dx = 3, \int_4^6 f(x)\,dx = 5$, and $\int_2^6 g(x)\,dx = -2$. Compute the following integrals, justifying your answers:

(a) $\int_2^4 3f(x)\,dx$?

(b) $\int_2^6 f(x) - g(x)\,dx$?

(c) $\int_6^4 f(x) - 3\,dx$?

Solution:

(a) $\int_2^4 3f(x)\,dx = 3\int_2^4 f(x)\,dx = 9$.

(b) $\int_2^6 f(x) - g(x)\,dx = \int_2^4 f(x)\,dx + \int_4^6 f(x)\,dx - \int_2^6 g(x)\,dx = 3 + 5 + 2 = 10$.

(c) $\int_6^4 f(x) - 3\,dx = \int_4^6 3\,dx - \int_4^6 f(x)\,dx = 6 - 10 = -4$.

Problem 2. We want to find $\frac{d}{dx}\int_{3x}^{x^3} \sqrt[3]{x+1}\,dx$. Unfortunately we can't apply the Fundamental Theorem of Calculus directly.

(a) This integral has variables in both the upper and lower bounds. Can you split it into multiple integrals, each of which has only one variable in a bound?

(b) To use the FTC we need the variable as the *upper* bound of each integral. How can we do that?

(c) Now for each integral you have set up, carefully take the derivative, paying attention to the chain rule.

(d) Combine this work to answer the original question.

Solution:

(a) We have
$$\int_{3x}^{x^3} \sqrt[3]{x+1}\,dx = \int_{3x}^{0} \sqrt[3]{x+1}\,dx + \int_{0}^{x^3} \sqrt[3]{x+1}\,dx.$$
Note that the choice of constant doesn't matter; you can pick anything there.

(b)
$$\int_{3x}^{0} \sqrt[3]{x+1}\,dx = -\int_{0}^{3x} \sqrt[3]{x+1}\,dx.$$

(c)
$$\frac{d}{dx}\int_{0}^{3x} \sqrt[3]{x+1}\,dx = \sqrt[3]{3x+1}\cdot 3$$
$$\frac{d}{dx}\int_{0}^{x^3} \sqrt[3]{x+1}\,dx = \sqrt[3]{x^3+1}\cdot 3x^2$$

(d)
$$\frac{d}{dx}\int_{3x}^{x^3} \sqrt[3]{x+1}\,dx = \sqrt[3]{x^3+1}\cdot 3x^2 - \sqrt[3]{3x+1}\cdot 3.$$

Problem 3. Compute the following integrals:

(a) $\displaystyle\int x(x+1)\,dx$

Solution: $\int x(x+1)\,dx = \int x^2 + x\,dx = \frac{x^3}{3} + \frac{x^2}{2} + C.$

We always like polynomials when we can have them.

(b) $\displaystyle\int x\sqrt{x}\,dx$

Solution: $\int x\sqrt{x}\,dx = \int x^{3/2} = \frac{x^{5/2}}{5/2} + C = \frac{2x^2\sqrt{x}}{5} + C.$

Writing things as pure fractional exponents almost always makes them easy to deal with.

(c) $\displaystyle\int 5\csc(x)\cot(x)\,dx$

Solution:

$$\int 5\csc(x)\cot(x)\,dx = -5\csc(x) + C.$$

This expression looks complicated but it's secretly not.

(d) $\int (x^4 - x)(x^2 + x + 1)\,dx.$

Solution:

$$\begin{aligned}\int (x^4 - x)(x^2 + x + 1)\,dx &= \int x^8 + x^5 + x^4 - x^3 - x^2 - x\,dx \\ &= \frac{1}{9}x^9 + \frac{1}{6}x^6 + \frac{1}{5}x^5 - \frac{1}{4}x^4 - \frac{1}{3}x^3 - \frac{1}{2}x^2 + C.\end{aligned}$$