

Math 1231: Single-Variable Calculus 1
George Washington University Spring 2023
Recitation 14

Jay Daigle

April 28, 2023

Problem 1. Find the average value of the function $\frac{x}{(x^2 + 1)^2}$ for $1 \leq x \leq 3$.

Problem 2. A 12in spring is stretched to 15in by a force of 75lbs.

- (a) What is the spring constant? What units does it have?
- (b) What is the function that gives force as a function of position? what units does it have?
- (c) What is the work done by stretching the spring from 16in to 20in? What units are your answer in?

Problem 3. We want to find the x -coordinate of the center of mass of the semicircle bounded by $y = \sqrt{16 - x^2}$ and $y = 0$, between $x = -4$ and $x = 4$.

- (a) Sketch a picture of this region. Geometrically, where should the center of mass be?
- (b) Sketch out the vertical strips you want to cut it into. What is the formula for the area of a strip? (Your formula should involve a dx term.)
- (c) Set up an integral to compute the area of the whole region? Can you do this antiderivative? Can you compute the area another way?
- (d) The *moment* of a strip along the x -axis is given by the formula $m_y = xA$. (No, that y is not a typo; it's a moment *along* the x -axis but *around* the y -axis.) What is a formula for the moment of a strip?

- (e) Use an integral to calculate the total moment of the region. Then divide by the area to get the center of mass.

Problem 4. Let's find the volume of the solid generated by rotating the region bounded by $y = x$ and $y = \sqrt{x}$ about the line $y = 1$.

- (a) Sketch a picture of the region, and draw in the line of revolution.
- (b) Lightly try to sketch what the solid of revolution will look like. Can you describe it in words?
- (c) Sketch in the slices you're going to use. Write down a formula for the volume of one slice. (This should involve a dx).
- (d) Set up an integral that computes the volume of the whole solid.
- (e) Compute the volume.