Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 2
Due Tuesday, January 31

This week's mastery quiz has two topics. You should definitely submit work on topic M1. You may or may not need to submit work on topic S1. If you got a score of 2 on topic S1 on last week's quiz, you do should not submit it again. You can check your current recorded scores on Blackboard.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 1: Estimation

Name:

Recitation Section:

Major Topic 1: Computing Limits

Compute the following limits, justifying your answers with clean and correct work.

(a) $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} =$

Solution:

$$\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3} = \lim_{x\to 3} \frac{x-3}{(x-3)(\sqrt{x+1}+2)} = \lim_{x\to 3} 1\sqrt{x+1}+2 = \frac{1}{4}.$$

(b) $\lim_{x\to 2} \frac{x^2+x-5}{3-x} =$

Solution:

$$\lim_{x\to 2} \frac{x^2+x-5}{3-x} = \frac{1}{1} = 1.$$

(c) $\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} =$

Solution:

$$\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x\to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x\to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x\to 1} \frac{1}{x} = 1.$$

Secondary Topic 1: Estimation

(a) Suppose $f(x) = \sqrt{x+1}$, and we want an output of approximately 3. What input a should we aim for? Find a δ so that if our input is $a \pm \delta$ then our output will be $3 \pm .5$. Explain how you found this δ and why it should give us what we want.

Solution: We want an input of about $a = 8$. By solving the equation we can see that if

$$\begin{aligned} f(x) &= 2.5 \\ x+1 &= 2.5^2 = 6.25 \\ x &= 5.25 \end{aligned} \qquad \begin{aligned} f(x) &= 3.5 \\ x+1 &= 3.5^2 = 12.25 \\ x &= 11.25 \end{aligned}$$

so we want x in $(5.25, 11.25)$. This ranges from $8-2.75$ to $8+3.25$, so we take $\delta = 2.75$ as the smaller of these two distances.

(b) We want to build a ramp that's eight times long as it is tall, and we want it to reach a height of 10 meters. Find a formula for δ in terms of ϵ, so that if the error in the *length* is less than δ then the error in the height is less than ϵ. Make sure your formula gives the **largest δ possible**, and justify your answer.

Solution: The height is $L/8$, so our output error is $|L/8-10| = \frac{1}{8}|L-80|$, which we want to be less than ε. So we get

$$\begin{aligned} |L/8 - 10| = \frac{1}{8}|L-80| &< \varepsilon \\ |L-80| &< 8\varepsilon. \end{aligned}$$

So if we take $\delta = 8\varepsilon$, then whenever the error in the length of our ramp $|L-80|$ is less than δ, then the error in our height should be less than ε.