Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 4
Due Tuesday, February 14

This week's mastery quiz has three topics. Everyone should submit work on S2 and M2. If you already have a 4/4 on M1, which would require you to have gotten full credit both the previous two weeks, you should not submit it.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Major Topic 2: Computing Derivatives
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

Compute the following limits, justifying your answers with clean and correct work.

(a) $\lim_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1} =$

Solution:

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} &= \lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} \\ &= \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.\end{aligned}$$

(b) $\lim_{x\to -\infty} \dfrac{3x^2+2x+1}{\sqrt{x^4-x^2+x}} =$

Solution:

$$\begin{aligned}\lim_{x\to -\infty} \frac{3x^2+2x+1}{\sqrt{x^4-x^2+x}} &= \lim_{x\to -\infty} \frac{3+2/x+1/x^2}{1/\sqrt{x^4}\sqrt{x^4-x^2+x}} \\ &= \lim_{x\to -\infty} \frac{3+2/x+1/x^2}{\sqrt{1-1/x^2+1/x^3}} \\ &= \frac{3+0+0}{\sqrt{1-0+0}} = 3.\end{aligned}$$

(c) $\lim_{x\to -2} \dfrac{x-2}{(x+2)^2} =$

Solution: The top approaches -4 and the bottom approaches 0, so

$$\lim_{x\to -2} \frac{x-2}{(x+2)^2} = \pm\infty.$$

Further, we see that the top is negative and the bottom is always positive, so in fact the limit is $-\infty$.

Major Topic 2: Computing Derivatives

Compute the following derivatives, using any tools we've developed in class.

(a) $\dfrac{d}{dx} \sec\left(\dfrac{x^2+1}{\sqrt{x^3-2}}\right) =$

Solution:

$$\sec\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\tan\left(\frac{x^2+1}{\sqrt{x^3-2}}\right)\frac{2x\sqrt{x^3-2}-(x^2+1)\frac{1}{2}(x^3-2)^{-1/2}\cdot 3x^2}{x^3-2}.$$

(b) $\dfrac{d}{dx}\tan^{3/5}\left(\sec\left(x^3-4\right)\right)$

Solution:

$$\frac{3}{5}\tan^{-2/5}\left(\sec\left(x^3-4\right)\right)\sec^2\left(\sec\left(x^3-4\right)\right)$$
$$\cdot\sec\left(x^3-4\right)\tan\left(x^3-4\right)3x^2$$

Secondary Topic 2 Definition of Derivative

Compute the following derivatives, *directly from the formal definition of derivative.*

(a) If $f(x)=3x^2-4x$, find $f'(-3)$.

Solution:

$$\begin{aligned}
f'(-3) &= \lim_{h\to 0}\frac{f(-3+h)-f(-3)}{h}\\
&= \lim_{h\to 0}\frac{3(h-3)^2-4(h-3)-39}{h}\\
&= \lim_{h\to 0}\frac{3h^2-18h+27-4h+12-39}{h}\\
&= \lim_{h\to 0}\frac{3h^2-22h}{h}\\
&= \lim_{h\to 0}3h-22=-22.
\end{aligned}$$

(b) If $g(x)=\frac{x}{x+2}$, find $g'(x)$.

Solution:

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{x+h}{x+h+2} - \frac{x}{x+2}}{h} \\
&= \lim_{h\to 0} \frac{(x+h)(x+2) - x(x+h+2)}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{x^2 + hx + 2x + 2h - x^2 - xh - 2x}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{2h}{h(x+2)(x+h+2)} \\
&= \lim_{h\to 0} \frac{2}{(x+2)(x+h+2)} = \frac{2}{(x+2)^2}.
\end{aligned}$$