Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 6
Due Tuesday, February 28

This week's mastery quiz has three topics. Everyone should submit work on M3 and S3. If you already have a 4/4 on M2, you should not submit it. But if Blackboard doesn't say you're at a 4/4, then you should submit again for another try.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Major Topic 3: Linear Approximation
- Secondary Topic 3: Rates of Change

Name:

Recitation Section:

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Major Topic 2: Computing Derivatives

Compute the following derivatives, using any tools we've developed in class.

(a) Compute $\frac{d}{dx}\tan^{3/5}\left(\csc\left(x^3-4\right)\right)$

Solution:

$$\frac{3}{5}\tan^{-2/5}\left(\csc\left(x^3-4\right)\right)\sec^2\left(\csc\left(x^3-4\right)\right)$$
$$\cdot(-1)\csc\left(x^3-4\right)\cot\left(x^3-4\right)3x^2$$

(b) Compute $\frac{d}{dx}\frac{\sin(\sec(x^2+1))}{x^4+\cos(x)} =$

Solution:

$$\frac{\big(\cos(\sec(x^2+1))(\sec(x^2+1)\tan(x^2+1))2x\big)(x^4+\cos(x))-(4x^3-\sin(x))\sin(\sec(x^2+1))}{(x^4+\cos(x))^2}.$$

Major Topic 3: Linear Approximation

(a) Give a formula for a linear approximation of $f(x) = x\sqrt{x+1}$ near the point $a = 3$.

Solution:

$$f'(x) = \sqrt{x+1} + \frac{x}{2\sqrt{x+1}}$$
$$f'(3) = 2 + \frac{3}{4} = \frac{11}{4}$$
$$f(x) \approx f(a) + f'(a)(x-a) = 6 + \frac{11}{4}(x-3).$$

(b) Use your answer in part (a) to estimate $f(2.8)$.

Solution: $f(2.8) \approx 6 + \frac{11}{4}(-.2) = 6 - \frac{11}{20} = \frac{109}{20}$.

(c) Write the equation for the tangent line to $g(x) = 2x - \tan(x)$ at the point $a = \pi$.

Solution:

$$g(\pi) = 2\pi - 0 = \pi$$
$$g'(x) = 2 - \sec^2(x)$$
$$g'(\pi) = 2 - 1 = 1$$
$$y = 2\pi + (x - \pi)$$

Secondary Topic 4: Rates of Change

(a) The *area moment of inertia* of a steel beam measures how difficult it is to bend, and is measured in m^4. If a square beam has a side length of s meters, then its moment of inertia is given by $L(s) = s^4/12$.

(i) What are the units of $L'(s)$? What does it represent physically? What does it mean if L' is big?

Solution: $L'(s)$ has units of $\frac{m^4}{m} = \mathrm{m}^3$. It describes how much increasing the side length by a meter would increase the area moment of inertia. If it's large, that means that making the beam a little longer will increase the moment of inertia by a lot.

(ii) Compute $L'(2)$. What does this tell you physically? What physical observation could you make to check your calculation?

Solution: $L'(s) = 4s^3/12 = s^3/3$ so $L'(2) = 8/3 = 8/3\mathrm{m}^3$. This tells us that if we increase the side length by one meter from 2 meters to 3 meters, we should increase the moment of inertia by about $8/3\mathrm{m}^4$.

(b) Suppose the vertical position of a weight on a spring in inches is given as a function of time in seconds by the formula $h(t) = \cos(2t)$.

(i) When is the velocity zero?

Solution: $p'(t) = -2\sin(2t)$ so the velocity is zero when $\sin(3t) = 0$. This happens when $2t = 0, \pi, 2\pi, \dots$, and thus when $t = 0, \pi/2, \pi, 3\pi/2, \dots$. In other words, at $n\pi/2$.

(ii) When is the acceleration zero?

Solution: $p''(t) = -4\cos(2t)$ is zero when $2t = \pi/2, 3\pi/2, \dots$, and thus when $t = \pi/4, 3\pi/4, 5\pi/4, \dots$. We could say $t = (2n+1)\pi/4$.