Math 1231: Single-Variable Calculus 1
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Recitation 6

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Problem 1 (Geometric Series)**.** Another function it's sometimes important to approximate is the "geometric series" formula $f(x) = \dfrac{1}{1-x}$, near $x = 0$.

(a) What is $f'(x)$?

(b) Find a linear approximation for $f(x)$ near $x = 0$.

(c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?

(d) Use your formula to estimate $\frac{1}{1.5}$ and $frac10.5$. Do these answers make sense?

(e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?

Now let's look at how we can *use* some of the formulas we've come up with.

Problem 2. (a) Let $g(x) = \frac{1}{1-x^2}$. We can linearly approximate this near $x = 0$; what do we get?

(b) But we can do this a different way. If we keep $f(x) = \frac{1}{1-x}$ the geometric series formula, we can view this function as $f(x^2)$. What does our approximation from problem 1 tell us that $f(x^2)$ is?

(c) Which of these will work better? Why? What makes (b) different from (a)?

Problem 3. We can do the same thing with another function. Let's take $h(x) = \frac{1}{1+x}$.

(a) We can do a straightforward linear approximation of h near 0. What do we get?

(b) We can also use our geometric series formula $f(x) = \frac{1}{1-x} \approx 1 - x$ here. Can we rewrite $h(x)$ in terms of f, like we did for g above? What does that give us as an approximation?

(c) There's one more way we can approximate this function. Remember the *binomial approximation* formula $(1+x)^\alpha \approx 1 + \alpha x$. How does that apply here, and what do we get?

Problem 4. (a) Use the binomial approximation to estimate $\sqrt{2}$ and $\sqrt[n]{2}$.

(b) Use the binomial approximation to estimate $\sqrt{17}$. (Remember: 17 is not close to 1! You need to be slightly clever here.)

(c) Can you find a formula to approximate $(1+x^n)^\alpha$ for a real number α?

(d) What does this tell us about $\sqrt{1+x^2}$?

Problem 5 (Bonus)**.** Find a formula to approximate $f(x) = x^3 + 3x^2 + 5x + 1$ near $a = 0$. What do you notice? Why does that happen?

Problem 6. Suppose a particle has height as a function of time given by $h(t\mathrm{s}) = (2t^3 - 3t^2 - 12t + 3)\,\mathrm{m}$.

(a) What is the velocity of this particle at time $t = 0$? What are the units, and why?

(b) What is the acceleration of this particle at time $t = 0$? What are the units and why?

(c) When is the particle speeding up? When is it slowing down?

Problem 7. Suppose we have a function $C(t)$ that tells us the concentration of a drug in the bloodstream as a function of time. Specifically, if we give t as the number of hours since the drug has been taken, C is the concentration in milligrams per liter.

(a) Write down the formula for the *definition of the derivative* of C, at time t_0. What are the units in this formula?

(b) What are the units of the derivative $C'(t)$?

(c) What does the derivative tell us, and why?