

Math 1231 Spring 2023
Single-Variable Calculus 1 Section 12
Mastery Quiz 6
Due Tuesday, February 28

This week's mastery quiz has four topics. Everyone should submit work on M3 and S4. If you already have a 4/4 on M2, you should not submit it. But if Blackboard doesn't say you're at a 4/4, then you should submit again for another try. If you already have a 2/2 on Rates of Change, you shouldn't submit it.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 2: Computing Derivatives
- Major Topic 3: Linear Approximation
- Secondary Topic 3: Rates of Change
- Secondary Topic 4: Related Rates

Name:

Recitation Section:

Major Topic 2: Computing Derivatives

(a) Find a formula for y' in terms of x and y if $x^3y + x^2y^2 + y^4 = 0$.

(b) Compute $\frac{d}{dx}g(x) = \left(\frac{x \csc(x)}{\sqrt{x^3 - x}}\right)^3$

Major Topic 3: Linear Approximation

(a) Write a tangent line to the curve $x^2y^2 = 5 + x + y$ at the point $(1, 3)$.

(b) Use linear approximation to estimate $\sqrt[4]{14}$.

Secondary Topic 3: Rates of Change

(a) The force a magnet exerts on a piece of iron depends on the distance between the magnet and the metal. Let $F(d) = \frac{2}{d^2}$ give the force exerted by the magnet in Newtons, where d is the distance between them in meters.

(i) What are the units of $F'(d)$? What does it $F'(d)$ represent physically? What would it mean if $F'(d)$ is big?

(ii) Calculate $F'(2)$. What does this tell you physically? What physical observation could you make to check your calculation?

(b) Suppose the distance between two particles in centimeters is given as a function of time in seconds by the formula $d(t) = t + \frac{1}{t}$.

(i) When is the velocity zero?

(ii) When is the acceleration zero?

Secondary Topic 4: Related Rates

A rocket is taking off with a perfectly vertical path, and is being tracked by a radar station on the ground four miles from the launch pad. We want to know how fast the rocket is rising when it is three miles high and its distance from the radar station is increasing at a rate of 3000 miles per hour.

- (a) Choose an equation to use for this problem, and explain why you chose that equation.
- (b) Use calculus to answer the question. Make sure you answer with a complete sentence.