Math 1231: Single-Variable Calculus 1
George Washington University   Spring 2023
Recitation 8

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March 10, 2023

Problem 1. Consider the function $f(x) = x^3 - 3x^2 + 1$ on $[-1, 4]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they?

Problem 2. Let's find the global extrema of $g(x) = \sqrt[3]{x^3 + 3x^2}$ on the closed interval $[-3, 2]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they?

Problem 3. Let $f(x) = |x^3 - 3x|$ on the closed interval $[-1, 3]$.

(a) Does this function have absolute extrema? Why?

(b) What are the critical points of this function?

(c) How many absolute extrema are there? What are they, and where are they?

Problem 4. We talked about using the combination of the Intermediate Value Theorem and Rolle's Theorem to figure out exactly how many solutions an equation has.

Let $g(x) = x^5 + x^3 + x - 1$. This is a function which has no roots you can actually write down in a useful way; it's a theorem that you can't give a nice algebraic description of the solutions to $x^5 + x^3 - 1 = 0$. But we can say some things about them.

(a) First, we want to get an idea of how many solutions we expect. Try plugging some small, easy numbers into this function, until you think you understand the function. How many solutions should it have?

(b) Now we want to prove we have to have a solution. Write up an argument in terms of the Intermediate Value Theorem to prove that $x^5 + x^3 - 1 = 0$ has a solution.

(c) Now we'll prove that there can't be any more. Check that this function satisfies the conditions of Rolle's Theorem. How does that prove we can't have two solutions?

(d) Can you explain informally why the equation can only have one solution, in terms of rates of change?

(e) Bonus: could you make the same argument about $g_2(x) = x^5 + x^3 - 1$? Why or why not? What would change?

Problem 5. We can also use the mean value theorem to constrain the possible values for a function. For instance, suppose I have a function f, and all I know is that $f(1) = 10$ and $f'(x) \geq 2$ for every x. I want to know about $f(4)$.

(a) What would this mean in English? How should we think about this physically? What does that tell you about $f(4)$?

(b) Use the Mean Value Theorem to set up an equation relating $f(1), f(4)$, and $f'(x)$. What does it tell you about $f(4)$?

(c) How do those two arguments relate to each other?

(d) is it possible for $f(4) = -30$?

Problem 6. Suppose $|g'(x)| \leq 2$ for all x, and $g(0) = 7$. We want to know about $g(5)$?

(a) What would this mean in English? How should we think about this physically? What does that tell you about $g(5)$?

(b) Use the Mean Value Theorem to set up an equation relating $g(0), g(5)$, and $g'(x)$. What does it tell you about $f(4)$?

(c) How do those two arguments relate to each other?

(d) is it possible for $g(5) = -30$?