

Math 1231 Spring 2023  
Single-Variable Calculus 1 Section 12  
Mastery Quiz 9  
Due Tuesday, March 28

This week's mastery quiz has three topics. Everyone should submit work on M4 and S5. If you already have a 4/4 on M3, you should not submit it. But if Blackboard doesn't say you're at a 4/4, then you should submit again for another try.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 3: Linear Approximation
- Major Topic 4: Optimization
- Secondary Topic 5: Curve Sketching

**Name:**

**Recitation Section:**

**Major Topic 3: Linear Approximation**

- (a) Find an equation for the line tangent to the curve  $3x^2y + 5xy^2 = 2x$  at the point  $(1, -1)$ .
- (b) Find a linear approximation to the function  $f(x) = \sin(x) \cos(x)$  near the point  $a = \pi/3$  and use it to approximate  $f(\pi/2)$ .

**M4: Extrema and Optimization**

(a) Classify the critical points and relative extrema of  $g(x) = \frac{2x - 1}{x^2 + 2}$ .

(b) Find the absolute extrema of  $g(x) = x^3 - 3x^2 - 9x + 3$  on  $[-2, 4]$ , and justify your claim that these are in fact absolute extrema.

## S5: Curve Sketching

Let  $f(x) = \frac{(x-2)^2}{x-1}$ . We can compute that

$$f'(x) = \frac{x(x-2)}{(x-1)^2}$$

$$f''(x) = \frac{2}{(x-1)^3}.$$

Sketch a graph of  $f$ . Your answer should discuss the domain, roots, asymptotes, limits at infinity, critical points and values, intervals of increase and decrease, and concavity and points of inflection.