

Math 1231: Single-Variable Calculus 1
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Recitation 9

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Problem 1. Let $g(x) = x \tan(x)$. We want to sketch a graph of g .

- (a) What is the domain of g ?
- (b) For simplicity, let's just look at $[-\pi/2, \pi/2]$. What can you say about any asymptotes it has?
- (c) Does this function have any roots you can find?
- (d) $g'(x) = \frac{\sin(x) \cos(x) + x}{\cos^2(x)}$. What are the critical points?
(Hint: when is $\sin(x) \cos(x)$ positive and when is it negative?)
- (e) What are the critical values?
- (f) Where is g increasing and decreasing? Does it have maxima or minima?
- (g) $g''(x) = 2 \sec^2(x)(1 + x \tan(x))$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?
- (h) Sketch the graph.

Problem 2. Let $f(x) = 2x^3 + 3x^2 - 36x$.

- (a) Find the critical points of f .
- (b) Which of these points can you classify using the second derivative test?

- (c) Classify all the critical points using the first derivative test.

Problem 3. Let $h(x) = \frac{x+2}{x-1}$. We want to sketch a graph of h .

- (a) What is the domain of h ? What can you say about any asymptotes it has?
- (b) Does this function have any roots? Where?
- (c) What happens as x approaches $+\infty$? $-\infty$?
- (d) $h'(x) = -3(x-1)^{-2}$. What are the critical points and values?
- (e) Where is h increasing and decreasing? Does it have maxima or minima?
- (f) $h''(x) = 6(x-1)^{-3}$. Where are the potential points of inflection? Where is h concave up and down?
- (g) Sketch the graph.

Problem 4. Let $g(x) = x^5 - 4x^3 + 4x + 7$. We want to sketch a graph of g .

- (a) What is the domain of g ? What can you say about any asymptotes it has?
- (b) Does this function have any roots you can find?
- (c) What happens as x approaches $+\infty$? $-\infty$?
- (d) $g'(x) = 5x^4 - 12x^2 + 4$. What are the critical points?
(Hint: if we set $u = x^2$ this becomes a quadratic, and we can factor it.)
- (e) What are the critical values?
(Hint: $g(x) = 7 + x(x^4 - 4x^2 + 4) = 7 + x(u^2 - 4u + 4)$.)
- (f) Where is g increasing and decreasing? Does it have maxima or minima?
- (g) $g''(x) = 20x^3 - 24x$. Where are the potential points of inflection, and what are their values? Where is h concave up and down?
- (h) Sketch the graph.