Math 1231 Practice Midterm Solutions

Instructor: Jay Daigle

- These are the instructions you will see on the real test, next week. I include them here so you know what to expect.
- You will have 75 minutes for this test.
- You are not allowed to consult books or notes during the test, but you may use a one-page, one-sided, handwritten cheat sheet you have made for yourself ahead of time.
- You may not use a calculator.
- This test has seven questions, over five pages. **You should not answer all seven questions.**
  - The first two problems are two pages, representing topics M1 and M2. You should do both of them, and they are worth 30 points each.
  - The remaining six problems represent topics S1 through S6. You will be graded on your best three, with a few possible bonus points if you also do well on the other two.
  - Doing three secondary topics well is much better than doing five poorly.
  - If you perform well on a question on this test it will update your mastery scores. Achieving a 27/30 on a major topic or 9/10 on a secondary topic will count as getting a 2 on a mastery quiz.

Problem 1 (M1)**.** Compute the following using methods we have learned in class. Show enough work to justify your answers.

(a) Find the tangent line to $h(x) = \arcsin(e^x)$ at $\ln(1/2)$.

(b) $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx =$

(c) $\displaystyle\int \frac{\cos(x)\sin(x)}{1+\cos^4(x)}\,dx =$

Problem 2 (M2)**.** Compute the following integrals using methods we have learned in class. Show enough work to justify your answers.

(a) $\displaystyle\int \frac{2x+1}{\sqrt{x^2-1}}\,dx$

(b) $\displaystyle\int x\sec^2 x\,dx$

(c) $\displaystyle\int_0^1 \frac{3x^2-6x+1}{(x^2-x-1)(x-2)}\,dx$

Problem 3 (S1)**.** Let $f(x) = \sqrt[3]{x^5 + x^4 + x^3 + x^2 + 2x}$. Find $(f^{-1})'(4)$.

Problem 4 (S2)**.** Find $\lim_{x\to 0} \frac{2\sin(x)-\sin(2x)}{x-\sin(x)}$.

Problem 5 (S3)**.** Use Simpson's rule and six intervals to estimate $\int_0^6 x^4\,dx$. Give an upper bound for the error on this approximation.

Problem 6 (S4)**.** Compute $\displaystyle\int_1^{10} \frac{1}{\sqrt[3]{x-2}}\,dx$.

Problem 7 (S5)**.** Find the surface area of the surface obtained by rotating $y = \sqrt{5 + 4x}$ for $-1 \leq x \leq 1$ about the x-axis.