Math 1232 Spring 2023
Single-Variable Calculus 2 Section 12
Mastery Quiz 11
Due Tuesday, April 18

This week's mastery quiz has three topics. You should definitely submit M4. If you have a 4/4 on M3, or 2/2 on S8, you don't need to submit it.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 3: Series Convergence
- Major Topic 4: Power and Taylor Series as functions
- Secondary Topic 8: Power Series

Name:

Recitation Section:

M3: Series Convergence

Analyze the convergence of the following three series. (Specify if they converge absolutely, converge conditionally, or diverge.)

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n}{n^3+n}$

(b) $\displaystyle\sum_{n=4}^{\infty} \frac{(-1)^n}{n^2/5+3n}$

(c) $\displaystyle\sum_{n=1}^{\infty} ne^{-n^2+1}$

M4: Taylor Series

(a) Let $f(x) = \cos^2(x)$. Use *the definition of a Taylor series* to find $T_4(x, \pi)$ for this function. (That is, find the terms up through the degree four term.)

(b) In class we computed a Taylor series for $\sin(x)$ centered at zero. Use the degree-seven Taylor polynomial to approximate $\sin(3) \approx T_7(3, 0)$. (You don't need to numerically simplify this.)

Using the Taylor series remainder, find an upper bound for the error in this approximation.

(c) Using series we already know, write down a formula for the (infinite) Taylor series for $x^3 e^{x^5/4}$, and then write down the first four non-zero terms of this series.

S8: Power Series

(a) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{(5x-3)^n}{\sqrt{n}}$.

(b) Find the radius of convergence and the interval of convergence of $\sum_{n=0}^{\infty} \frac{n}{5^n}(x-3)^n$.