Math 1232: Single-Variable Calculus 2
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Recitation 11

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Problem 1 (Bessel Function)**.** The Bessel function (of order 0) is critical to any physics done in cylindrical coordinates, and thus any physics that occurs on a cylinder. We saw it earlier as the solution to the differential equation $x^2y'' + xy' + x^2y = 0$, but it can also be given by the power series:

$$J_0(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2^{2n}(n!)^2}.$$

What is the radius of convergence? What is the interval of convergence?

Problem 2. What is the interval of convergence of

$$\sum_{n=0}^{\infty} \frac{n^2(x-1)^n}{7^{n+2}}?$$

Problem 3. Consider the function $f(x) = \dfrac{1}{1+x^6}$.

(a) Could you compute $\displaystyle\int \frac{1}{1+x^6}\,dx$? How?

(b) Does it help if I tell you that $1 + x^6 = (1 + x^2)(x^2 - \sqrt{3}x + 1)(x^2 + \sqrt{3}x + 1)$?

(c) Now write a power series for $f(x)$ centered at 0. What is the interval of convergence?

(d) Compute the integral of your power series. What is the interval of convergence there?

Problem 4. We want to compute $\int_3^4 \frac{1}{1-(x-4)^3}\,dx$

(a) Find a power series for to compute $\frac{1}{1-(x-4)^3}$.

(b) Integrate the power series from 3 to 4. Does this converge?

(c) Sum the first five terms to estimate $\int_3^4 \frac{1}{1-(x-4)^3}\,dx$.

(d) Use an online integral calculator to find the integral. How close is your answer to the true answer?