

Math 1232 Spring 2023  
Single-Variable Calculus 2 Section 12  
Mastery Quiz 12  
Due Tuesday, April 25

This week's mastery quiz has **four** topics. You should definitely submit M4, S9, and S10. If you have a 4/4 on M3, you don't need to submit it.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 3: Series Convergence
- Major Topic 4: Power and Taylor Series as functions
- Secondary Topic 9: Applications of Taylor Series
- Secondary Topic 10: Parametrization

**Name:**

**Recitation Section:**

### M3: Series Convergence

Analyze the convergence of the following three series. (Specify if they converge absolutely, converge conditionally, or diverge.)

(a)  $\sum_{n=1}^{\infty} (-1)^n \frac{3n^4 - 1}{n^5 + 1}$ .

(b)  $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2 5^n}$ .

(c) Analyze the convergence of  $\sum_{n=1}^{\infty} \frac{n! + 1}{(n + 1)!}$ .

### M4: Taylor Series

(a) If  $f(x) = \sum_{n=0}^{\infty} \frac{3^n}{n!} (x + 2)^n$ , compute  $\frac{d}{dx} f(x)$  and  $\int f(x) dx$ .

(b) Using series we already know, write down a formula for the (infinite) Taylor series for  $(1 + 3x)^{2/3}$ , and then write down the degree-three polynomial explicitly.

(c) Find an upper bound for the error if you use  $T_3(x) = x - \frac{x^2}{2} + \frac{x^3}{3}$  to approximate  $g(x) = \ln(1 + x)$  at  $x = .5$ .

### S9: Applications of Taylor Series

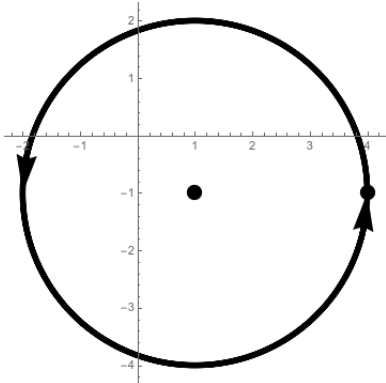
(a) Use a Taylor series to compute  $\lim_{x \rightarrow 0} \frac{\cos(x^2) - 1 + x^4/2}{x^8} =$

(b) Use a degree-three Taylor polynomial to estimate  $\sqrt{1.2}$ .

(c) Use a degree-five Taylor polynomial to estimate  $\arctan(.1)$ .

### S10: Parametrization

(a) Find a parametrization for the circle of radius 3 centered at  $(1, -1)$ , starting at  $(4, -1)$  and going **counterclockwise twice** around the circle.



- (b) Find a parametrization of the ellipse  $x^2/4 + y^2 = 1$ . (Hint: what are the  $x$  and  $y$  intercepts?)
- (c) Find a parametric equation for the line tangent to the curve  $x = 1 + \sqrt{t}, y = t^3$  at the point  $(2, 1)$ .