

Math 1232: Single-Variable Calculus 2
George Washington University Spring 2023
Recitation 12

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Problem 1. Let's find the Taylor series of $f(x) = e^x$ centered at $a = 1$.

- (a) Compute f', f'', f''' . Find a formula for $f^{(n)}(x)$.
- (b) Give a formula for $T_f(x, 1)$.
- (c) We want to know if $f(x) = T_f(x, 1)$. Find a formula for $R_k(x, 1)$. Can you show this goes to 0 as k goes to infinity?
- (d) We already have another power series for f :

$$T_f(x, 0) = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

You should have a different power series; but can you convince yourself it *should* give the same function? (What happens if you plug $x - 1$ into this series?)

Problem 2. Let's do something silly, and compute the Taylor series of a polynomial.

- (a) Let $f(x) = x^3 + 3x^2 + 1$. Find the Taylor series centered at zero. Was that what you expected?
- (b) Now find the Taylor series centered at 2. Do you get the same thing? What's useful about this?

Problem 3. Back in section 2 we talked about the bell curve function $p(x) = e^{-x^2}$. (Technically we should be talking about $\frac{1}{\sqrt{2\pi}}e^{-x^2/2}$ but that's annoying and doesn't change the details enough to be interesting.)

- (a) Find a power series for $p(x)$ centered at zero. (This should not require any real calculations.)
- (b) Find an antiderivative for $p(x)$, using power series.
- (c) Write down a series that computes $\int_0^1 p(x) dx$.
- (d) Add up the first three or four terms of this series. What do you get? Can you estimate the error in this calculation?

Problem 4. In class we worked out a Taylor series for $g(x) = \ln(x)$ centered at $a = 1$:

$$T_g(x, 1) = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} (x - 1)^n.$$

But is this actually equal to $g(x)$?

- (a) Write down a formula for $R_k(x, 1)$.
- (b) Compute $T_5(2, 1)$. Can you estimate the error?
- (c) Compute $T_5(1.5, 1)$. Can you estimate the error?
- (d) Compute $T_5(0, 1)$. Can you estimate the error here?
- (e) What would you need to assume to show this goes to zero as k goes to infinity? Does that make sense?