Math 1232 Spring 2023
Single-Variable Calculus 2 Section 12
Mastery Quiz 2
Due Tuesday, January 31

This week's mastery quiz has two topics. You should definitely submit work on topic M1. You may or may not need to submit work on topic S1. If you got a score of 2 on topic S1 on last week's quiz, you do should not submit it again. You can check your current recorded scores on Blackboard.

Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material. Feel free to consult your notes, but please don't discuss the actual quiz questions with other students in the course.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Tuesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Calculus of Transcendental Functions
- Secondary Topic 1: Invertible Functions

Name:

Recitation Section:

M1: Calculus of Transcendental Functions

(a) Compute $\dfrac{d}{dx} x^{\ln(x)}$.

Solution: The simplest approach is to use logarithmic differentiation.

$$\begin{aligned} y &= x^{\ln(x)} \\ \ln(y) &= \ln(x)\ln(x) = \ln(x)^2 \\ \frac{y'}{y} &= 2\ln(x)\frac{1}{x} \\ y &= \frac{2\ln(x)}{x} y = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

Alternatively, we could compute

$$\begin{aligned} \frac{d}{dx} x^{\ln(x)} &= \frac{d}{dx}\left(e^{\ln(x)}\right)^{\ln(x)} = \frac{d}{dx} e^{\ln(x)^2} \\ &= e^{\ln(x)^2} \cdot 2\ln(x)\frac{1}{x} = \frac{2\ln(x)x^{\ln(x)}}{x}. \end{aligned}$$

(b) Compute $\displaystyle\int_0^{\ln(5)} e^x\sqrt{1+3e^x}\,dx$.

Solution: Set $u = 1+3e^x$, so $du = 3e^x\,dx$, and we see that $u(0) = 4$ and $u(\ln(5)) = 16$.

$$\begin{aligned} \int_0^{\ln(5)} e^x\sqrt{1+3e^x}\,dx &= \int_4^{16} \frac{1}{3}\sqrt{u}\,du \\ &= \frac{2}{9}u^{3/2}\Big|_4^{16} = \frac{2}{9}\cdot 64 - \frac{2}{9}\cdot 8 = \frac{112}{9}. \end{aligned}$$

Alternatively, we could substitute our integral back:

$$\begin{aligned} \int e^x\sqrt{1+3e^x}\,dx &= \int \frac{1}{3}\sqrt{u}\,du \\ &= \frac{2}{9}u^{3/2} + C = \frac{2}{9}(1+3e^x)^{3/2} + C. \\ \int e^x\sqrt{1+3e^x}\,dx &= \frac{2}{9}(1+3e^x)^{3/2}\Big|_0^{\ln(5)} \\ &= \frac{2}{9}\cdot 64 - \frac{2}{9}\cdot 8 = \frac{112}{9}. \end{aligned}$$

(c) Compute $\displaystyle\int \frac{\sin(2t)+\cos(2t)}{\sin(2t)-\cos(2t)}\,dt$.

Solution: We can take $u = \sin(2t) - \cos(2t)$. Then $du = (2\cos(2t) + 2\sin(2t))dt$, and we get

$$\begin{aligned}\int \frac{\sin(2t)+\cos(2t)}{\sin(2t)-\cos(2t)}\,dt &= \int \frac{\sin(2t)+\cos(2t)}{u}\frac{du}{2\cos(2t)+2\sin(2t)} \\ &= \int \frac{1}{2}u\,du = \frac{1}{2}\ln|u| + C \\ &= \frac{1}{2}\ln|\sin(2t)-\cos(2t)| + C.\end{aligned}$$

S1: Invertible Functions

(a) Find a formula for the inverse of $g(x) = (x-1)^3 + 3$.

Solution:

$$\begin{aligned} y &= (x-1)^3 + 3y - 3 & &= (x-1)^3 \\ \sqrt[3]{y-3} &= x - 1 \\ x &= 1 + \sqrt[3]{y-3} \end{aligned}$$

so $g^{-1}(y) = 1 + \sqrt[3]{y-3}$. (You can use whichever variable you like in your formula.)

(b) Let $h(x) = x^5 + x$. Compute $(h^{-1})'(2)$.

Solution: By the Inverse Function Theorem, we know that

$$(h^{-1})'(2) = \frac{1}{h'(h^{-1}(2))}.$$

Guess and check shows that $h(1) = 2$ so $h^{-1}(2) = 1$. And we know that

$$h'(x) = 5x^4 + 1$$

and thus

$$h'(1) = 5 + 1 = 6$$

Thus

$$(h^{-1})'(2) = \frac{1}{6}.$$

(c) Compute $e^{5\ln(3)-2\ln(4)}$. (Give an exact answer with no decimals.)

Solution:

$$e^{5\ln(3)-2\ln(4)} = (e^{\ln(3)})^5/(e^{\ln(4)})^2 \qquad = \frac{3^5}{4^2} = \frac{243}{16}.$$