Math 1232: Single-Variable Calculus 2
George Washington University Spring 2023
Recitation 2

Jay Daigle

January 27, 2023

Problem 1. (a) Compute $\sin(\arctan(5))$.

(b) Compute $\dfrac{d}{dx}\arccos(\sqrt{x})$

(c) Compute $\dfrac{d}{dx}\arctan(x+\sec(x))$

Problem 2. Compute the following integrals:

(a) $\displaystyle\int \frac{\arcsin(x)}{\sqrt{1-x^2}}\,dx.$

(b) $\displaystyle\int_0^1 \frac{e^{2x}}{1+e^{4x}}\,dx.$

(c)

Problem 3. (a) In class, we saw that $\lim_{x\to+\infty}\frac{\ln(x)}{x}=0$. What is $\lim_{x\to+\infty}\frac{\ln(x^2)}{x}$?

(b) Compute $\lim_{x\to+\infty}\frac{\ln(x^n)}{x}$ for $n>0$.

(c) Compute $\lim_{x\to+\infty}\frac{\ln(x)}{x^\epsilon}$ for $\varepsilon>0$.

(d) What do parts (a-c) tell you about the relationship between polynomials and $\ln(x)$?

(a) In class we saw that $\lim_{x\to+\infty}\frac{e^x}{x}=+\infty$. Compute $\lim_{x\to+\infty}\frac{e^x}{x^2}$.

(b) Compute $\lim_{x\to+\infty}\frac{e^x}{x^n}$ for $n>0$.

(c) What do parts (e-f) tell you about the relationship between e^x and polynomials?

Problem 4. (a) We want to compute $\lim_{x\to\pi/2} \sec(x) - \tan(x)$.

(b) Can we use L'Hospital's Rule on this as written? Can we change it to a form where L'Hospital's Rule works?

(c) What is the limit?

$$\begin{aligned}\lim_{x\to\pi/2} \sec(x) - \tan(x) &= \lim_{x\to\pi/2} \left(\frac{1}{\cos(x)} - \frac{\sin(x)}{\cos(x)}\right) \\ &= \lim_{x\to\pi/2} \frac{1-\sin(x)^{\nearrow 0}}{\cos(x)_{\searrow 0}} \\ &=^{\text{L'H}} \lim_{x\to\pi/2} \frac{-\cos(x)^{\nearrow 0}}{-\sin(x)_{\searrow 1}} = \frac{0}{1} = 0.\end{aligned}$$

(d) Now let's compute $\lim_{x\to 0} \cot(2x)\sin(6x)$.

(e) Can we rewrite this so we can apply L'Hospital's Rule?

(f) What is the limit?

Problem 5. Let's compute $\lim_{x\to 0^+} x^{\frac{1}{\ln(x)-1}}$

(a) What indeterminate form is this?

(b) If $y = x^{\frac{1}{\ln(x)-1}}$, what is $\ln|y|$?

(c) Compute $\lim_{x\to 0^+} \ln|y|$.

(d) Compute $\lim_{x\to 0^+} x^{\frac{1}{\ln(x)-1}}$.