

Math 1232: Single-Variable Calculus 2
George Washington University Spring 2023
Recitation 5

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Problem 1. We want to find $\int \frac{x^5 + x - 1}{x^3 + 1} dx$.

- (a) What's the first tool we need to apply here? (Hint: not partial fractions!)
- (b) Once we get it in a more manageable form, things should simplify out nicely. What is the final integral?

Problem 2. We've looked briefly at the integral $\int \frac{1}{1 + e^x} dx$. Let's try it again with our new tools.

- (a) Try the substitution $u = e^x$. What do you get? What tools can apply to the result?
- (b) Do a partial fractions decomposition to get the integral.

Problem 3 (Bonus). Let's see if we can work out the integral of secant! This isn't at all obvious.

- (a) We want $\int \sec(x) dx = \int \frac{1}{\cos(x)} dx$. Since this is a fraction, we can multiply the top and bottom through by $\cos(x)$. This makes the expression more complicated, but it does allow us to use a trig identity. What do we get?
- (b) Now we can do a u substitution. What u substitution seems reasonable? Does it help us at all?
- (c) Now we can use partial fractions to finish the problem off. We wind up with an awkward answer, but an answer.

- (d) The most common formula for the integral of $\sec(x)$ is $\ln |\sec(x) + \tan(x)| + C$. Is that the same as what you got? (Hint: use logarithm laws and multiplication by the conjugate.)

Problem 4 (Bonus). What if we want to find $\int \frac{x^4 + 6x^3 + 4x^2 + 8x + 11}{(x-1)^2(2x+1)(x^2+4x+5)} dx$?

Problem 5. We want to find the cross-sectional area of a two-meter-long airplane wing. We measure its width every 20 centimeters, and get: 5.8, 20.3, 26.7, 29.0, 27.6, 27.3, 23.8, 20.5, 15.1, 8.7, 2.8. Use the trapezoidal rule and Simpson's rule to estimate the area of the wing.

Problem 6. Consider the function $f(x) = x^2 + 1$.

- (a) Use the trapezoid rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (b) Use the midpoint rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (c) Use Simpson's rule with six intervals to estimate $\int_{-4}^2 f(x) dx$.
- (d) Which of these do you expect to be most accurate? Which do you expect to be least accurate?
- (e) Compute $\int_{-4}^2 f(x) dx$. What do you find? Why?

Problem 7. Let $g(x) = e^{-x^2}$, and suppose we want to compute $\int_{-1}^2 e^{-x^2} dx$, and get the answer correct to two decimal places.

- (a) We can compute that $g''(x)$ varies between -2 and $.9$ when x is in $[-1, 2]$. What value should we take for K ?
- (b) How many subintervals should we use to get the answer correct to within two decimal places using the trapezoid rule?
- (c) How many subintervals should we use to get the answer correct to within two decimal places using the midpoint rule?
- (d) We can compute that $g'''(x)$ varies between -8 and 12 . What value should we take for L ?
- (e) How many subintervals should we use to get the answer correct to within two decimal places using Simpson's rule?