Math 1232: Single-Variable Calculus 2

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Recitation 6

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Problem 1. We want to compute $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$.

(a) Can you compute an antiderivative? Can you evaluate it at 0 and 2?

(b) Did part (a) finish the problem? Sketch a picture of the graph. What should we be concerned about?

(c) Carefully set up a computation that will find $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$. (Hint: It should have two limit operations in it.)

(d) What did we learn from this that we didn't learn from (a)?

Problem 2. We want to figure out if $\int_0^{+\infty} e^{-x^2}\,dx$ converges—that is, if it's finite or infinite.

(a) If we can find an antiderivative, we can just compute the improper integral directly. Why doesn't that work?

(b) Since we can't integrate this directly we might want to use a comparison test. We need to find an easy-to-integrate function that's larger than e^{-x^2}. Find a function $f(x)$ that makes $f(x)e^{-x^2}$ easy to integrate.

(c) If $f(x) \geq 1$, then we can just integrate $f(x)e^{-x^2}$. Is it?

(d) This is where we can pull in a trick. Is there some a where $f(x) > 1$ when $x > a$? (You may need to adjust your $f(x)$ here, especially the sign. It's fine as long as you can still integrate it.)

(e) We know $\int_a^{+\infty} e^{-x^2}\,dx \leq \int_a^{+\infty} f(x)e^{-x^2}\,dx$. Compute the new improper integral; is it finite?

(f) Now we just have to deal with $\int_0^a e^{-x^2}\,dx$. We can't do that integral exactly, but that's fine: you should be able to tell whether it's finite or not without doing any calculations. How?

(g) Does $\int_0^{+\infty} e^{-x^2}\,dx$ converge?

Problem 3. Consider the graph of the hyperbola $xy = 1$ as x varies from 1/3 to 1, and y varies from 1 to 3. We want to find the arc length of this curve.

(a) The obvious choice is to write this as a function of x. What would the function be there?

(b) Set up an integral to compute the arc length writing y as a function of x.

(c) Alternatively, we could write this as a function of y. What would that function be?

(d) Set up an integral to compute the arc length writing x as a function of y.

(e) Are these integrals the same? Can you compute them?

(f) Plug the integrals you set up into an integral calculator. Do they get the same answer?

Problem 4 (Gabriel's Trumpet/Infinite Paint Can)**.** Consider a trumpet-shaped container, given by taking the curve $y = 1/x$ and rotating around the x-axis, for $x \geq 1$. We're going to imagine this as a giant, oddly-shaped paint can. (See figure 0.1.)

(a) First let's find the volume of this infinite object. For this we want to take cross-sections perpendicular to the x-axis. Each cross-section will be a circle. Find a formula for the radius of this circle, given the x value.

(b) Set up an integral to compute the volume. This will be an improper integral. Remember that we need to integrate an *area*, so the integrand should be the area of the circular cross-section.

(c) Compute the integral. What does that integral tell you about this object? How much paint could we pour into it?

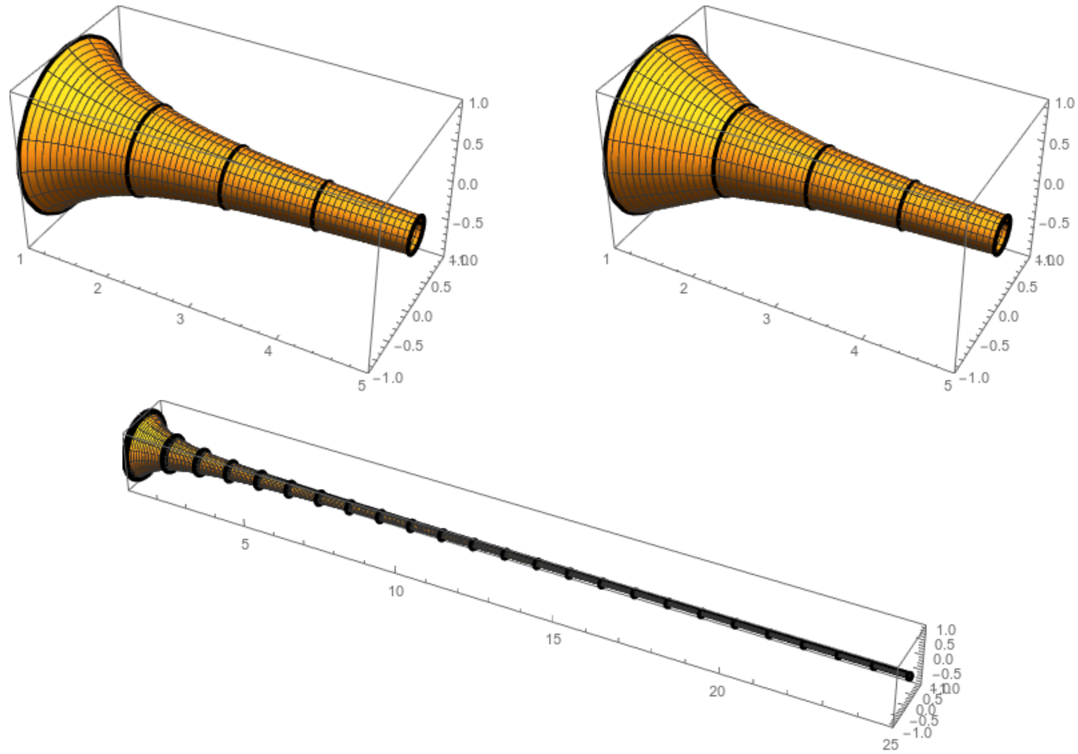

Figure 0.1: Gabriel's Trumpet

(d) Now we want to find the surface area. We already found a radius, so we just need to set up an (improper) integral to compute the surface area.

(e) Compute that integral. What is the surface area? How much paint would we need to paint the inside of this object?

(f) Do your answers to (c) and (e) make sense together?