Math 1232: Single-Variable Calculus 2
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Recitation 7

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Problem 1. Suppose we have a Hooke's Law system of a weight on a spring. Suppose $m = k$, so that we get the differential equation $x''(t) = -x(t)$.

(a) From class, we know the general solution to this differential equation. What is it?

(b) Suppose now we start (at time 0) with the weight stationary and displaced by 1 meter. What initial conditions does this correspond to?

(c) Find the specific solution to this initial value problem.

(d) What does this describe physically? Does that solution make physical sense?

Problem 2. We can also do a different setup that is not technically an initial value problem. Suppose we have a Hooke's Law setup with a weight on a spring, and $m = k$, so $x(t) = a\sin(t) + b\cos(t)$.

(a) Suppose the weight starts with a displacement of 2, and at time $t = \pi/4$ the displacement is $\sqrt{8}$. How can we encode that mathematically?

(b) Why is this not an initial value problem? (We call this a "boundary value problem". Why do you think we call it that?)

(c) Is this enough information to find a specific solution to the differential equation? What is it?

Problem 3. (a) Can you find a (non-trivial) solution to $x''(t) = -2x(t)$? Make sure to check that your solution is right.

(b) Can you find a (non-trivial) solution to $x''(t) = 2x(t)$? Make sure to check that your solution is right.

Problem 4. Find the general solution to $(y^2 + xy^2)y' = 1$.

Problem 5 (Evans Price Change Model)**.** In this problem we want to create an economic model of how prices change over time. Let's imagine we're studying a market where people buy and sell pairs of headphones.

(a) First we need to understand supply and demand curves. We can write a demand function $D(p)$ that takes in a price p and tells us the number of people who would want to buy a pair of headphones at that price on a given day. Should this function have a positive or negative derivative, and why?

(b) We can also write a supply function $S(p)$ that takes in a price p, and tells us the number of pairs of headphones that suppliers are willing to sell at that price each day. Should this function have a positive or negative derivative, and why?

(c) Suppose we have $D(p) = 80 - 5p$ and $S(p) = 5p - 20$. What will be the equilibrium price, where the quantity demanded (number of headphones people want to buy) is the same as the quantity supplied. How many headphones will be sold each day?

(d) That gives us the equilibrium price. But people actually trading in the market don't know the equilibrium price; it takes actual time to find it. We want to study the function $p(t)$, which tells us the price as a function of time.

The Evans model of price change says that the price changes at a rate proportional to the difference between the quantity demanded and the quantity supplied. That is, if a lot more people want to buy than sell, the price will rise quickly; if only a few people more people want to buy than sell, the price will still rise, but slowly.

Spend a couple minutes trying to write down a differential equation that encodes this model. Talk to your neighbors, see if they agree!

(e) If you didn't already, make sure your equation only has p as a variable. But there should also be a constant k. What does k represent?

(f) Let's assume $k = 1$. Do you recognize this equation? Can you find the general form of the solution?

(g) Does the trivial solution $p(t) = 0$ solve this system? Does that make sense?

(h) What is the limit as $t \to +\infty$? Does that make sense?

(i) Can you find a specific solution when $p(0) = 15$?

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Example 0.1. Confirm that $f(x) = x^2 + x + 1$ satisfies $2f(x) - xf'(x) = x + 2$.

We compute $f'(x) = 2x + 1$, so $2f(x) - xf'(x) = 2x^2 + 2x + 2 - (2x^2 + x) = x + 2$.