

Math 1232: Single-Variable Calculus 2
George Washington University Spring 2023
Recitation 8

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March 10, 2023

Problem 1. Let $(a_n) = (-6, 4, \frac{-8}{3}, \frac{16}{9}, \frac{-32}{27}, \dots)$.

- (a) Find a closed-form formula for a_n .
- (b) Is there a real function f so that $f(n) = a_n$?
- (c) What is $\lim_{n \rightarrow \infty} a_n$? Why?

Problem 2 (Factorials). (a) What is $4!$? What is $\frac{4!}{3!}$?

- (b) What is $\frac{5!}{4!}$? What is $\frac{5!}{3!}$?
- (c) Can you figure out what $\frac{202!}{200!}$ is?

Problem 3. (a) Compute $\lim_{n \rightarrow \infty} \frac{n}{n!}$. Justify your answer.

- (b) Compute $\lim_{n \rightarrow \infty} \frac{e^n}{n!}$.
- (c) Now compute $\lim_{n \rightarrow \infty} \frac{n^k}{n!}$, where $k > 0$ is an integer.

Problem 4. Consider the sequence $(a_n) = (\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots)$.

- (a) We don't have a closed-form formula for this sequence, but we can still say things about it. What happens if we square each element of the sequence, and then divide by 2?

- (b) We want to find the limit of this sequence. Half of this is easy: *if* the sequence converges, we can use a trick to find the limit.

Suppose $\lim_{n \rightarrow \infty} a_n = L$. What can you say about $L^n/2$?

- (c) Can you figure out what L is, if the limit exists?
- (d) That all relied on the idea that the limit existed. We want to use completeness to prove this. We need to show this sequence is increasing and bounded above.

If $0 \leq x \leq 2$, explain why $x \leq \sqrt{2x}$.

- (e) If $0 \leq x \leq 2$, explain why $\sqrt{2x} \leq 2$.

- (f) How does this prove the limit exists?

Problem 5. The discrete equivalent of a derivative is a *difference quotient*. Given a function $f(n)$ defined on positive integers, we can define $\Delta f(n) = f(n+1) - f(n)$.

- (a) Does that look like a derivative? What pieces are missing, and why?
- (b) If $f(n) = n^2$, compute $\Delta f(n)$. Compute $f'(n)$. How are they related?
- (c) If $g(n) = \frac{1}{n}$, compute $\Delta g(n)$. Compute $g'(n)$. How are they related?