Derivatives Practice Sheet Solutions

Compute the following derivatives, for practice. Do not simplify!

(a)

$$\sqrt[5]{\frac{x^2\sin(3x)}{\tan(x)}}$$

$$\frac{1}{5}\left(\frac{x^2\sin(3x)}{\tan(x)}\right)^{-4/5}\frac{(2x\sin(3x)+x^2\cos(3x)3)\tan(x)-\sec^2(x)x^2\sin(3x)}{\tan^2(x)}$$

(b)

$$\tan^4(\sqrt[3]{x^5+x^3+2}+1).$$

$$\begin{aligned}\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1) &= 4\tan^3(\sqrt[3]{x^5+x^3+2}+1)\cdot\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot(\sqrt[3]{x^5+x^3+2}+1)'\\ &= 4\tan^3(\sqrt[3]{x^5+x^3+2}+1)\sec^2(\sqrt[3]{x^5+x^3+2}+1)\\ &\quad\cdot\left(\frac{1}{3}(x^5+x^3+1)^{-2/3}\cdot(5x^4+3x^2)\right).\end{aligned}$$

(c)

$$\cos\left(\frac{x^2-\sqrt{5x^2+1}}{x^4+\sin(x/2)}\right)$$

$$-\sin\left(\left(\frac{x^2-\sqrt{5x^2+1}}{x^4+\sin(x/2)}\right)\cdot\right.$$
$$\left.\frac{(2x-\frac{1}{2}(5x^2+1)^{-1/2}10x)(x^4+\sin(x/2))-(4x^3+\cos(x/2)\frac{1}{2})(x^2-\sqrt{5x^2+1})}{(x^4+\sin(x/2))^2}\right)$$

(d)

$$\left(\sin(4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})})\right)^5$$

$$\begin{aligned}&5\left(\sin(4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})})\right)^4\cdot\cos((4\sqrt{5\tan(\sqrt[3]{\csc(3x^2)})}))\cdot\\ &\quad 4\cdot\frac{1}{2}(5\tan(\sqrt[3]{\csc(3x^2)}))^{-1/2}\cdot 5\sec^2(\sqrt[3]{\csc(3x^2)})\cdot\\ &\quad\frac{1}{3}(\csc(3x^2))^{-2/3}\cdot(-1)\csc(3x^2)\cot(3x^2)\cdot 6x\end{aligned}$$

(e)

$$\frac{\tan(x)+\sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}}}{x^4\sin(x/5)\cos(x)}$$

$$\frac{\left(\sec^2(x)+\frac{1}{3}\left(\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}\right)^{-2/3}\cdot\frac{(7\sec(x)\tan(x)+1)\sqrt{2\sin(x)}-\frac{1}{2}(2\sin(x))^{-1/2}2\cos(x)(7\sec(x)+x)}{2\sin(x)}\right)(x^4\sin(x/5)\cos(x)}{x^8\sin^2(x/5)\cos^2(x)}$$

$$-\frac{(4x^3\sin(x/5)\cos(x)+x^4\cos(x/5)(1/5)\cos(x)-x^4\sin(x/5)\sin(x))\left(\tan(x)+\sqrt[3]{\frac{7\sec(x)+x}{\sqrt{2\sin(x)}}}\right)}{x^8\sin^2(x/5)\cos^2(x)}$$

(f)

$$\cot^{4/3}\left(\frac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right)$$

$$\frac{4}{3}\cot^{1/3}\left(\frac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right)\cdot(-1)\csc\left(\frac{\sqrt{\sin(3x/2)+1}+x}{\sin(7x^{5/4})}\right)$$

$$\cdot\frac{\left(\frac{1}{2}(\sin(3x/2)+1)^{-1/2}\cos(3x/2)3/2+1\right)\sin(7x^{5/4})-\cos(7x^{5/4})(7\cdot\frac{5}{4}x^{1/4})\left(\sqrt{\sin(3x/2)+1}+x\right)}{\sin^2(7x^{5/4})}$$

(g)

$$\sqrt[7]{\csc\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)}$$

$$\frac{1}{7}\left(\csc\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)\right)^{-6/7}$$

$$\cdot(-1)\csc^2\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)\cdot\cot\left(\frac{\cos\left((x^3+1)^2\sin(x)\right)+\tan(x)}{\sqrt{5x+3\sin(x)}}\right)$$

$$\cdot\frac{(-\sin((x^3+1)^2\sin(x))(2(x^3+1)3x^2\sin(x)+(x^3+1)^2\cos(x))+\sec^2(x))\sqrt{5x+3\sin(x)}}{5x+3\sin(x)}$$

$$-\frac{\frac{1}{2}(5x+3\sin(x))^{-1/2}(5+3\cos(x))(\cos\left((x^3+1)^2\sin(x)\right)+\tan(x))}{5x+3\sin(x)}$$