Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 1
Due Wednesday, September 4

This week's mastery quiz has one topic. Please do your best on that topic. Don't worry if you make a minor error, but try to demonstrate your mastery of the underlying material.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Secondary Topic 1: Estimation

Name:

Recitation Section:

Secondary Topic 1: Estimation

(a) Suppose $f(x) = x^2 + 3x$, and we want an output of approximately 10. What input a should we aim for if we want $a > 0$? Find a δ so that if our input is $a \pm \delta$ then our output will be 10 ± 1. Justify your answer.

Solution: We want an input of about $a = 2$. Our output error will be $|x^2+3x-10| = |x+5| \cdot |x-2|$. We know that $x + 5 \approx 7 < 8$, so we have

$$|x^2 + 3x - 10| = |x + 5| \cdot |x - 2| < 8|x - 2| < 1$$

so we need $|x - 2| < 1/8$. So we can take $\delta = 1/8$.

(b) We want to amplify an electrical signal. Our amplifier will multiply the voltage by a factor of six, and we want an output signal of $24 \pm \varepsilon$ volts. Find a formula for δ in terms of ε, so that if the input error is less than δ then the error in the output is less than ε. Make sure your formula gives the **largest** δ **possible**, and justify your answer.

Solution: Our output error is $|6x - 24| = 6|x - 4|$, and we want this to be less than ε. So we get

$$\begin{aligned} |6x - 24| = 6|x - 4| &< \varepsilon \\ |x - 4| &< \varepsilon/6. \end{aligned}$$

So if we take $\delta = \varepsilon/6$, then whenever the error in our input voltage $|x - 4| < \delta$ then the error in our output voltage should be less than ε.