

Math 1231 Fall 2024  
Single-Variable Calculus I Section 12  
Mastery Quiz 10  
Due Wednesday, November 6

This week's mastery quiz has three topics. They are the same topics as last week. If you have a 4/4 in M3, or a 2/2 in S7 or S8, you don't need to submit them.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course.**

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

**Topics on This Quiz**

- Major Topic 3: Optimization
- Secondary Topic 7: Curve Sketching
- Secondary Topic 8: Physical Optimization

**Name:**

**Recitation Section:**

## Major Topic 3: Optimization

- (a) The function  $f(x) = \frac{x}{x^2 + 1}$  has absolute extrema either on the interval  $[0, 3]$  or on the interval  $(2, 4)$ . Pick one of those intervals, explain why  $f$  has extrema on that interval, and find the absolute extrema.
- (b) Classify the critical points and relative extrema of  $h(x) = \sin(x) + \cos(x)$  on  $[0, 2\pi]$ .

## Secondary Topic 7: Curve Sketching

Sketch the graph of  $g(x) = 3x^4 - 4x^3 - 36x^2 + 64 = (x + 2)^2(3x - 4)(x - 4)$ . We have  $g'(x) = 12x^3 - 12x^2 - 72x = 12x(x - 3)(x + 2)$  and  $g''(x) = 36x^2 - 24x - 72 = 12(3x^2 - 2x - 6)$ .

You should discuss the domain, limits, critical points, intervals of increase and decrease, concavity, and possible points of inflection.

## Secondary Topic 8: Physical Optimization

Suppose that a company that produces and sells  $x$  units of a product each day makes a revenue of  $R(x) = 260x - 9x^2/10$  dollars per day and has costs given by  $C(x) = 1000 + 100x + x^2/10$  dollars per day. What is the maximum profit that can be made (where profit is revenues minus costs)?