Math 1231: Single-Variable Calculus 1
George Washington University Fall 2024
Recitation 10

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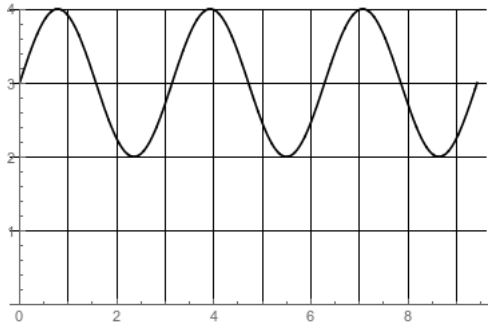
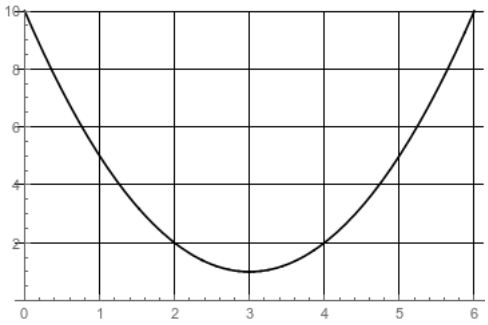
Problem 1. For the following curves, find an upper bound and a lower bound for the area under the curve, and then give your best estimate for the actual area.



(between 0 and 9; ignore the trailing bit off the right edge)



(between 0 and 6; ignore the trailing bit off the right edge)

Problem 2. Consider the function $f(x) = \sqrt{1-x^2}$ between $x = 0$ and $x = 1$.

(a) What shape is the graph? Draw a picture to look at for the rest of this.

(b) Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?

(c) Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?

(d) Find an upper bound using four rectangles.

(e) Find a lower bound using four rectangles.

(f) Can you guess what the area under the curve is exactly? (Hint: look at the graph.)

Problem 3. Consider the function $g(x) = x^3$ between $x = 0$ and $x = 1$.

(a) Estimate the area using two rectangles with right endpoints. Is this an upper bound, a lower bound, or neither?

(b) Estimate the area using two rectangles with left endpoints. Is this an upper bound, lower bound, or neither?

(c) Find an upper bound using four rectangles.

(d) Find a lower bound using four rectangles.

(e) Find a formula using right endpoints to estimate the area using n rectangles, in summation form.

(f) Use your summation rules to get a closed-form formula, with no summation signs in it.

(g) Take a limit to find the exact area under this curve. (Use your summation rules!)

Problem 4. Suppose we know that $\int_2^4 f(x)\,dx = 3$, $\int_4^6 f(x)\,dx = 5$, and $\int_2^6 g(x)\,dx = -2$. Compute the following integrals, justifying your answers:

(a) $\int_2^4 3f(x)\,dx$?

(b) $\int_2^6 f(x) - g(x)\,dx$?

(c) $\int_6^4 f(x) - 3\,dx$?

Problem 5. (a) Use the Fundamental Theorem of Calculus to compute $\frac{d}{dx}\int_2^x \sqrt{t^5 - t}\,dt$.

(b) Compute $\frac{d}{dx}\int_x^5 s^5 + \cos(s^2)\,ds$. What rule did you have to use here other than the FTC?

(c) Compute $\frac{d}{dx}\int_{-3}^{x^2} \sqrt{t^3 + 1}\,dt$. What rule did you have to use here other than the FTC?

Problem 6. We want to find $\frac{d}{dx}\int_{3x}^{x^3} \sqrt[3]{x+1}\,dx$. Unfortunately we can't apply the Fundamental Theorem of Calculus directly.

(a) This integral has variables in both the upper and lower bounds. Can you split it into multiple integrals, each of which has only one variable in a bound?

(b) To use the FTC we need the variable as the *upper* bound of each integral. How can we do that?

(c) Now for each integral you have set up, carefully take the derivative, paying attention to the chain rule.

(d) Combine this work to answer the original question.