Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 2
Due Wednesday, September 11

This week's mastery quiz has two topics. If you have a 2/2 on S1 (as shown on Blackboard), you don't need to submit it again, but if you have a 1/2 or 0/2 you should submit again. Everyone should submit on M1.

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 1: Estimation

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) Compute $\lim_{x\to 3} \dfrac{\sqrt{7-x}-2}{x-3} =$

Solution:

$$\lim_{x\to 3} \frac{\sqrt{7-x}-2}{x-3} = \lim_{x\to 3} \frac{7-x-4}{(x-3)(\sqrt{7-x}+2)} = \lim_{x\to 3} \frac{-1}{\sqrt{7-x}+2} = \frac{-1}{4}.$$

(b) Compute $\lim_{x\to 1} \dfrac{1}{x-1} - \dfrac{1}{x^2-x} =$

Solution:

$$\lim_{x\to 1} \frac{1}{x-1} - \frac{1}{x^2-x} = \lim_{x\to 1} \frac{x^2-x-(x-1)}{(x-1)(x^2-x)} = \lim_{x\to 1} \frac{(x-1)^2}{x(x-1)^2} = \lim_{x\to 1} \frac{1}{x} = 1.$$

(c) Compute $\lim_{x\to 0} \dfrac{\sin(3x)\sin(4x)}{x\sin(2x)} =$

Solution:

$$\begin{aligned}\lim_{x\to 0} \frac{\sin(3x)\sin(4x)}{x\sin(2x)} &= \lim_{x\to 0} \frac{\sin(3x)}{3x}\frac{\sin(4x)}{4x}\frac{12x^2}{x\sin(2x)} \\ &= \lim_{x\to 0} \frac{2x}{\sin(2x)}\frac{6x}{x} \\ &= \lim_{x\to 0} \frac{6x}{x} = 6.\end{aligned}$$

Secondary Topic 1: Estimation

(a) We can make a pound of charcoal from nine pounds of wood, and we want to produce about 8 pounds of charcoal. Find a formula for δ in terms of ϵ, so that if the error in the amount of *wood* is less than δ then the error in the amount of charcoal is less than ϵ. Make sure your formula gives the **largest δ possible**, and justify your answer.

Solution: The amount of charcoal we produce is $W/9$, so our output error is $|W/9 - 8| = \frac{1}{9}|W-72|$, which we want to be less than ε. So we get

$$\begin{aligned}|W/9 - 8| = \frac{1}{9}|W-72| &< \varepsilon \\ |W-72| &< 9\varepsilon.\end{aligned}$$

So if we take $\delta = 9\varepsilon$, then whenever the error in the amount of wood we use, $|W-72|$, is less than δ, then the error in our amount of charcoal should be less than ε.

(b) Suppose $f(x) = x^2 + 4x$, and we want an output of approximately 12. What input a should we aim for, if we want a positive input? Find a δ so that if our input is $a \pm \delta$ then our output will be 12 ± 1. Justify your answer.

Solution: We want an input of about $a = 2$. Our output error will be $|x^2+4x-12| = |x+6| \cdot |x-2|$. We know that $x + 6 \approx 8 < 9$, so we have

$$|x^2 + 4x - 12| = |x + 6| \cdot |x - 2| < 9|x - 2| < 1$$

so we need $|x - 2| < 1/9$. So we can take $\delta = 1/9$.