

Math 1231: Single-Variable Calculus 1  
George Washington University Fall 2024  
Recitation 2

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September 10, 2024

**Problem 1** (Warmup). Let  $f(x) = \frac{x^2 + \sin(x) + 3}{x^2 - x - 2}$ .

- (a) Where is  $f$  continuous? Where is it discontinuous?
- (b) What is  $\lim_{x \rightarrow 0} f(x)$ ?

**Problem 2.** Let  $f(x) = \frac{x-1}{x^2-1}$ .

- (a) What is  $f(2)$ ? Is  $f$  continuous at 2?
- (b) What is  $\lim_{x \rightarrow 2} f(x)$ ?
- (c) What is  $f(1)$ ? Is  $f$  continuous at 1?
- (d) What function can we find that's almost the same as  $f$ , but defined and continuous at 1? (Is this function the same as  $f$ ?)
- (e) What is  $\lim_{x \rightarrow 1} f(x)$ ?

**Problem 3.** Let  $g(x) = \frac{(x+1)^2 - 1}{x+2}$ .

- (a) Is  $g$  continuous where it's defined? Where is it undefined?
- (b) Can you find a function that's almost identical to  $g$  but continuous everywhere?
- (c) What is  $\lim_{x \rightarrow -2} g(x)$ ?

**Problem 4.** Let  $h(x) = \frac{x-1}{\sqrt{5-x}-2}$ .

- (a) Is this function continuous where it's defined? Where is it undefined?
- (b) We can factor an  $x - 1$  out of the top, but we can't obviously factor one out of the bottom. We need to use an algebraic trick make the  $x - 1$  appear. What tricks do we have that might work?
- (c) What is  $\lim_{x \rightarrow 1} h(x)$ ?

**Problem 5.** We want to compute  $\lim_{x \rightarrow 3} \frac{\sin(x^2-9)}{x-3}$ .

- (a) What rule do we know we need to invoke here?
- (b) What  $\theta$  are we going to need to pick for this to work out, and why?
- (c) Do algebra so that you can invoke the small angle approximation. What is the limit? (Are you using the AIF property?)
- (d) Go back to the beginning, and see what our heuristic idea that  $\sin(\theta) \approx \theta$  would have told you. Does that match with what you got?