Math 1231 Fall 2024
Single-Variable Calculus I Section 11
Mastery Quiz 2
Due Wednesday, September 11

This week's mastery quiz has two topics. Everyone should submit both. (Even if you got a 2 last week on M1, your score depends on your best two attempts.)

Feel free to consult your notes, but please **don't discuss the actual quiz questions with other students in the course**.

Remember that you are trying to demonstrate that you understand the concepts involved. For all these problems, justify your answers and explain how you reached them. Do not just write "yes" or "no" or give a single number.

Please turn this quiz in class on Wednesday. You may print this document out and write on it, or you may submit your work on separate paper; in either case make sure your name and recitation section are clearly on it. If you absolutely cannot turn it in in person, you can submit it electronically but this should be a last resort.

Topics on This Quiz

- Major Topic 1: Computing Limits
- Secondary Topic 2: Definition of Derivative

Name:

Recitation Section:

Major Topic 1: Computing Limits

(a) $\lim_{x\to -2} \frac{x-2}{(x+2)^2} =$

Solution: The top approaches -4 and the bottom approaches 0, so

$$\lim_{x\to -2} \frac{x-2}{(x+2)^2} = \pm\infty.$$

Further, we see that the top is negative and the bottom is always positive, so in fact the limit is $-\infty$.

(b) $\lim_{x\to +\infty} \frac{x^2-3x+2}{x^3+x-1} =$

Solution:

$$\lim_{x\to +\infty} \frac{x^2-3x+2}{x^3+x-1} = \lim_{x\to +\infty} \frac{1/x-3/x^2+2/x^3}{1+1/x^2-1/x^3} = \frac{0-0+0}{1+0-0} = 0.$$

(c) $\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} =$

Solution:

$$\begin{aligned}\lim_{x\to 1} \frac{\sqrt{x+3}-2}{x-1} &= \lim_{x\to 1} \frac{x+3-4}{(x-1)(\sqrt{x+3}+2)} \\ &= \lim_{x\to 1} \frac{1}{\sqrt{x+3}+2} = \frac{1}{4}.\end{aligned}$$

Secondary Topic 2: Definition of Derivative

(a) If $f(x) = \sqrt{x+3}$, find $f'(6)$, directly from the definition of derivative.

Solution:

$$\begin{aligned}f'(2) &= \lim_{h\to 0} \frac{f(6+h)-f(6)}{h} \\ &= \lim_{h\to 0} \frac{\sqrt{9+h}-3}{h} \\ &= \lim_{h\to 0} \frac{h}{h(\sqrt{9+h}+3)} \\ &= \lim_{h\to 0} \frac{1}{\sqrt{9+h}+3} \\ &= \lim_{h\to 0} \frac{1}{6}.\end{aligned}$$

(b) If $g(x) = \frac{1}{x+2}$, find $g'(a)$, explicitly using the definition of the derivative.

Solution:

$$\begin{aligned}
g'(x) &= \lim_{h\to 0} \frac{g(x+h) - g(x)}{h} \\
&= \lim_{h\to 0} \frac{\frac{1}{x+h+2} - \frac{1}{x+2}}{h} \\
&= \lim_{h\to 0} \frac{(x+2) - (x+h+2)}{(x+2)(x+h+2)h} \\
&= \lim_{h\to 0} \frac{-h}{(x+2)(x+h+2)h} \\
&= \lim_{h\to 0} \frac{-1}{(x+2)(x+h+2)} = \frac{-1}{(x+2)^2}.
\end{aligned}$$