Math 1231: Single-Variable Calculus 1

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Recitation 4

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Problem 1. Let $g(x) = \frac{1}{x+3}$.

(a) Write down a limit expression to compute $g'(2)$. Be careful with order of operations and parentheses!

(b) Now compute $g'(2)$.

(c) Write a limit expression to compute $g'(x)$. Again, make sure you get your order of operations right.

(d) Compute $g'(x)$.

Problem 2. Let $a(x) = |x|$ be the absolute value function.

(a) Write down a formula for a as a piecewise function.

(b) Write down a limit expression for the derivative of a at 0.

(c) What is the limit from the right?

(d) What is the limit from the left?

(e) What does that tell you about the derivative?

Problem 3. Let $f(x) = \sqrt{x^2 - 4}$.

(a) Set up a limit expression to calculate $f'(x)$. Do you think $h \to 0$ or $x \to a$ will be easier here?

(b) Compute $f'(x)$.

(c) Where is f differentiable? Where is it not differentiable?

Problem 4.   (a) Let $h(x) = \tan^2(x)$. Find functions f and g so that $h(x) = (f \circ g)(x)$.

(b) Compute $f'(x)$ and $g'(x)$. Use that info to compute $h'(x)$.

(c) Now let $h(x) = \tan(x^2)$. Find functions f and g so that $h(x) = (f \circ g)(x)$.

(d) Compute $f'(x)$ and $g'(x)$. Use that information to compute $h'(x)$.

Problem 5. Consider the function $\sec^2(x^2 + 1)$

(a) Find functions f and g so that $(f \circ g)(x) = \sec^2(x^2 + 1)$.

(b) Talk to the people next to you. Did they pick the same f and g that you did? Can you find a different pair of functions f and g that also work?

(c) Find functions f, g, h so that $(f \circ g \circ h)(x) = \sec^2(x^2 + 1)$.

(d) Compute f', g', and h'.

(e) What is $\frac{d}{dx} \sec^2(x^2 + 1)$?

Problem 6 (Bonus). Find
$$\frac{d}{dx} \frac{\sin(x^2) + \sin^2(x)}{x^2 + 1}$$

Problem 7 (Bonus).   (a) Compute

$$\frac{d}{dx} \sqrt{\frac{\sqrt{x} + 1}{(\cos x + 1)^2}}$$

(b) Find

$$\frac{d}{dx} \tan^4(\sqrt[3]{x^5 + x^3 + 2} + 1).$$

Problem 8 (Bonus). Calculate

$$\frac{d}{dx} \left(\frac{\sin^2\left(\frac{x^2+1}{\sqrt{x-1}}\right) + \sqrt{x^3 - 2}}{\cos(\sqrt{x^2 + 1} + 1) - \tan(x^4 + 3)} \right)^{5/3}$$