Math 1231-13: Single-Variable Calculus 1

George Washington University Fall 2024

Recitation 5

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Problem 1 (Bonus)**.** Consider the function $\sec^2\left(x^2+1\right)$

(a) Find functions f and g so that $(f \circ g)(x) = \sec^2\left(x^2+1\right)$.

(b) Talk to the people next to you. Did they pick the same f and g that you did? Can you find a different pair of functions f and g that also work?

(c) Find functions f, g, h so that $(f \circ g \circ h)(x) = \sec^2\left(x^2+1\right)$.

(d) Compute f', g', and h'.

(e) What is $\frac{d}{dx}\sec^2\left(x^2+1\right)$?

Problem 2. Find

$$\frac{d}{dx}\frac{\sin(x^2)+\sin^2(x)}{x^2+1}$$

Problem 3. (a) Compute

$$\frac{d}{dx}\sqrt{\frac{\sqrt{x}+1}{(\cos x+1)^2}}$$

(b) Find

$$\frac{d}{dx}\tan^4(\sqrt[3]{x^5+x^3+2}+1).$$

Problem 4 (Bonus)**.** Calculate

$$\frac{d}{dx}\left(\frac{\sin^2\left(\frac{x^2+1}{\sqrt{x}-1}\right)+\sqrt{x^3-2}}{\cos(\sqrt{x^2+1}+1)-\tan(x^4+3)}\right)^{5/3}$$

Problem 5 (Geometric Series)**.** One function it's sometimes important to approximate is the "geometric series" formula $f(x) = \frac{1}{1-x}$, near $x = 0$.

(a) What is $f'(x)$?

(b) Find a linear approximation for $f(x)$ near $x = 0$.

(c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?

(d) Use your formula to estimate $\frac{1}{1.5}$ and $frac10.5$. Do these answers make sense?

(e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?

Problem 6. (a) Use the binomial approximation to estimate $\sqrt{2}$ and $\sqrt[n]{2}$.

(b) Use the binomial approximation to estimate $\sqrt{17}$. (Remember: 17 is not close to 1! You need to be slightly clever here.)

(c) Can you find a formula to approximate $(1 + x^n)^\alpha$ for a real number α?

(d) What does this tell us about $\sqrt{1 + x^2}$?

Problem 7 (Bonus)**.** Find a formula to approximate $f(x) = x^3 + 3x^2 + 5x + 1$ near $a = 0$. What do you notice? Why does that happen?