Math 1231: Single-Variable Calculus 1
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Recitation 5

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Problem 1 (Geometric Series)**.** One function it's sometimes important to approximate is the "geometric series" formula $f(x) = \dfrac{1}{1-x}$, near $x = 0$.

(a) What is $f'(x)$?

(b) Find a linear approximation for $f(x)$ near $x = 0$.

(c) Use this formula to estimate $\frac{1}{.9}$ and $\frac{1}{1.01}$. Do these answers make sense?

(d) Use your formula to estimate $\frac{1}{1.5}$ and $frac10.5$. Do these answers make sense?

(e) Use your formula to estimate $f(-1)$ and $f(1)$. Do these answers make sense?

Problem 2. (a) Use the binomial approximation to estimate $\sqrt{2}$ and $\sqrt[n]{2}$.

(b) Use the binomial approximation to estimate $\sqrt{17}$. (Remember: 17 is not close to 1! You need to be slightly clever here.)

(c) Can you find a formula to approximate $(1 + x^n)^\alpha$ for a real number α?

(d) What does this tell us about $\sqrt{1 + x^2}$?

Problem 3 (Bonus)**.** Find a formula to approximate $f(x) = x^3 + 3x^2 + 5x + 1$ near $a = 0$. What do you notice? Why does that happen?

Problem 4. Suppose a particle has height as a function of time given by $h(ts) = (2t^3 - 3t^2 - 12t + 3)$ m.

(a) What is the velocity of this particle at time $t = 0$? What are the units, and why?

(b) What is the acceleration of this particle at time $t = 0$? What are the units and why?

(c) When is the particle speeding up? When is it slowing down?

Problem 5. Suppose that $p(t) = 10 - 2t$ is momentum (in kg m/s) of a ball thrown directly upwards, as a function of time (in seconds).

(a) What units does the derivative $p'(t)$ take as input? What units are its output? (Do you know of any physical quantity that's represented by those units?)

(b) What does the derivative $p'(t)$ represent physically? What would it mean for $p'(t)$ to be big, or small?

(c) Calculate $p'(3)$. What does this tell you? What physical observation could you measure to check if your calculation was correct?

Problem 6. Suppose the cost of buying m machines is $C(m) = 500 + 10m + .05m^2$. There's some start-up cost to having any machines at all; then each machine costs a bit more than the previous one.

(a) What are the units of the inputs to the function C? What are the units of the outputs?

(b) What is $C(1)$? $C(10)$? $C(100)$?

(c) Find a formula for $C'(m)$. What are the units of the input and output to $C'(m)$?

(d) What is $C'(10)$? How should we interpret this number?

(e) What is the *average* cost per machine when you have ten machines? How does this compare to your previous answer?

(f) What is $C''(m)$? What are the units? What is $C''(10)$ and how should we interpret it?

Problem 7 (Bonus)**.** Let $Q(p) = 10000 - 10p$ give the number of widgets you can sell at a given price p.

(a) If you set a price of \$100, how many widgets will you be able to sell? What if you set a price of \$1000?

(b) What is the derivative of Q? What are its units?

(c) What is $Q'(100)$ and what does that tell you?